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SHORT PERIOD VERTICAL OSCILLATIONS IN THE WESTERN BASIN OF THE NORTH ATLANTIC

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INTRODUCTION

Because of general interest in the subject of vertical oscillations in the sea and because such information is scanty for the ocean basins, an investigation of the question in the western North Atlantic was initiated by the establishment of "Atlantis" station 2639, July 9 to 13, 1936.

The significance of vertical oscillations in the sea has been known from the earlier work of Helland-Hansen and Nansen, and, in 1926, these authors summarized their conception of the problem as follows: "By earlier investigations we have found that there are probably considerable vertical oscillations of the water layers in various regions of the ocean. Hence the occasional vertical series of observations cannot be expected always to represent the average conditions at any particular station. It is therefore of great importance for the discussion of the general conditions in a sea-area on the basis of the observations made, to study how far these actual observations at the different stations and different depths may be regarded as representative." Also, in this same paper we find the statements: "It has already been mentioned that the oscillations described have obviously to a great extent some connection with the tides; but how the tidal wave can produce vertical movements of such dimensions in the different strata of the sea seems to us at present to be inexplicable. We have here a phenomena of fundamental importance to oceanography, which has to be made the subject of special methodical investigations."

The present discussion is based on the short period temperature, salinity, and oxygen variations which were observed to occur at various fixed levels at "Atlantis" station 2639 during a four and one quarter day period of observation, $9^h\ 30'$, July 9 to $15^h\ 45'$, July 13, 1936 (L.C.T.). In addition to a brief theoretical treatment, the results are here analyzed from a standpoint of practical oceanography, and, in so far as the observations warrant, the effect of short period oscillations on temperature, salinity, and oxygen distribution is discussed with certain of the more prominent results generally deduced from these factors.

It is necessary to bear in mind that the tentative conclusions from this investigation are based on the results of continuous observations that extended over a period of little more than four days, and frequently the details are confined to the more complete set of observations taken in the upper twelve hundred meters during the final 24 hour period, July 12 to 13, so that they are applicable to oceanographic investigations only in a very general way. It seems that while short period oscillations appear to be correlated with daily and half daily tidal periods, there are other factors which disturb the periodic recurrence of the observed phenomena in varying degrees of intensity. Hence, the results obtained during a twenty-four lunar hour period (the principle period considered for purposes of analysis) are not necessarily applicable to those of any other day. However, while the degree and nature of the variance with time is not known, we are not prevented from drawing certain qualitative conclusions for the region represented by station 2639, and, in some cases, the conclusions may be extended to the ocean as a whole. In this instance may be mentioned especially the order of magnitude of the effect of vertical displacements of the water column on dynamic calculations of total transport between two fixed verticals in the sea, as are generally calculated from temperature and salinity observations, and the discussion and tabulation of the probability of occurrence of various degrees of difference between random oxygen observations at the same fixed levels of the same region.

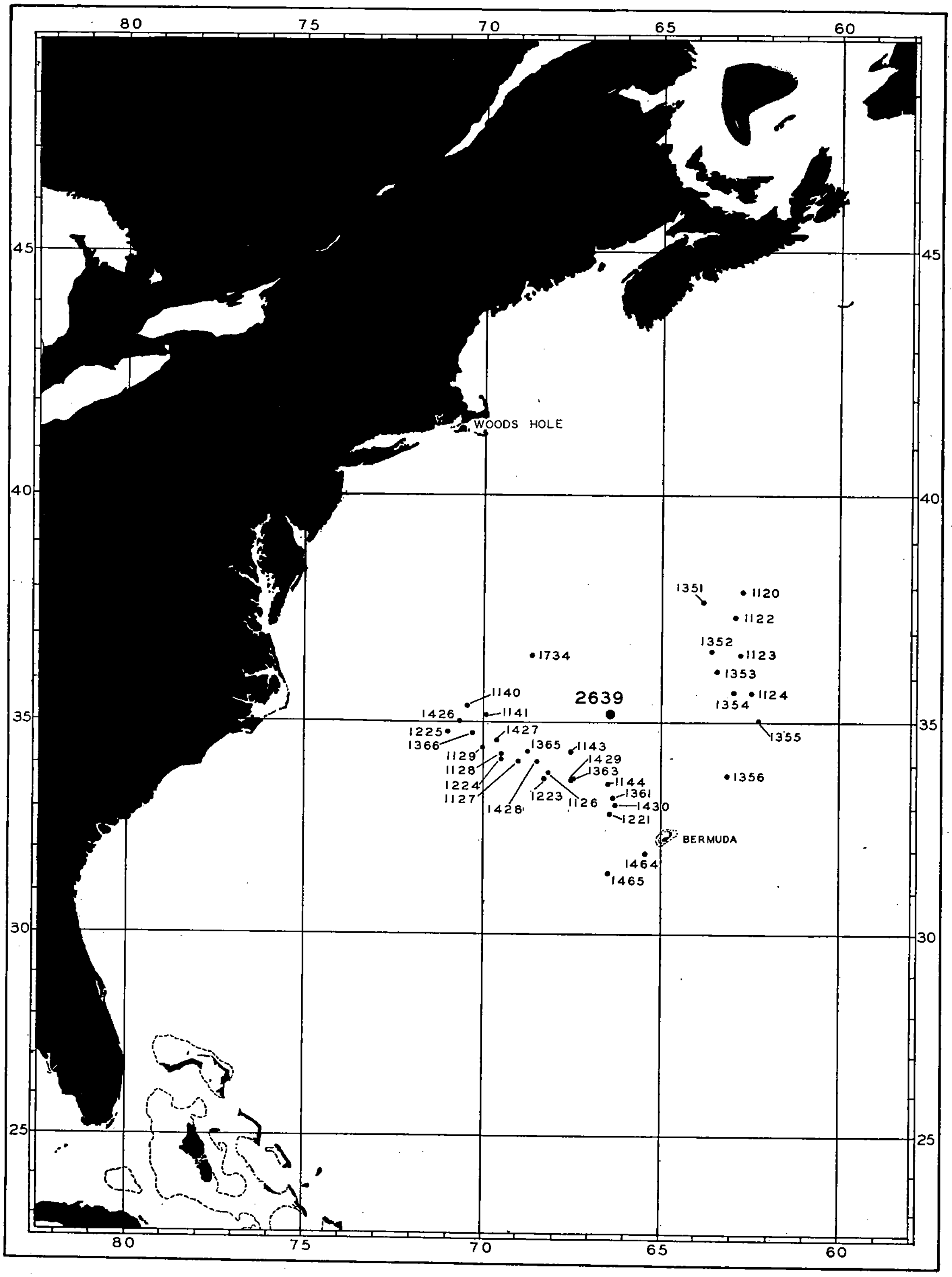

FIG. 1.—Location of "Atlantis" oceanographic stations referred to in this report.

THE STATION

For purposes of this investigation it was desirable to locate a station where horizontal gradients of hydrographic characteristics of the water mass should be as small as possible for considerable distance in all directions. Such a condition was attained by locating station 2639 at a position about 180 miles northwest of Bermuda (mean position: 66° 25′W, 35° 07′N); this being somewhat more than 150 miles from the eastern edge of the American coastwise convergence (Fig. 1). Horizontal gradients of the fundamental characteristics (temperature, salinity, oxygen, etc.) of the water are very small (page 16) and isolines representing these properties maintain relatively level positions in the oceanographic sections.[1]

The total period of actual operation at this station extended from 9^h 30′ on July 9 to 15^h 45′ on July 13, 1936. During this time the "Atlantis" was not anchored and occasional steaming back to position was necessary. Checks on the amount and direction of the ship's drift (aided by good weather conditions) were maintained by frequent astronomical observations by the ship's officers. Thirty-eight of the forty-four individual lowerings of water bottles and thermometers (90%) were made within a five mile radius of the mean position of the station. Of the remaining six lowerings, the three comprising series C and the first lowering of series D are located approximately 10 and 16 miles, respectively, northwest of the mean position, while series U and V (one lowering each) are located 7½ and 11 miles, respectively, to the north. The drift of "Atlantis" during the period of observation is summarized in table 1.

TABLE 1

Date July 1936	Time Interval		ΔT	ΔD	Direction	$\frac{\Delta D}{\Delta t}$	Lowerings
9	$08^h40'$	$12^h00'$	3.3	1.25	75°W	0.38	A_1, A_2, A_3
9	$18^h40'$	$20^h13'$	1.6	?	75°W	?	B_1, B_2, B_3
9–10	$20^h13'$	$04^h50'$	8.6	5.75	77°W	0.67	C_1, C_2, C_3
10	$04^h50'$	$06^h51'$	2.0	1.00	73°W	0.50	D_1
10	$09^h00'$	$12^h00'$	3.0	0.75	73°W	0.25	D_2
10	$12^h00'$	$16^h00'$	4.0	6.40	73°W	1.60	D_3
10	$17^h20'$	$20^h12'$	2.9	1.75	94°W	0.61	E_1, E_2
10–11	$20^h12'$	$04^h50'$	8.6	5.00	94°W	0.58	E_3, F_1, F_2, F_3
11	$04^h50'$	$06^h10'$	1.3	1.00	88°W	0.75	—
11	$07^h00'$	$12^h00'$	5.0	1.50	5°E	0.30	G_1, G_2, G_3
11	$13^h32'$	$20^h20'$	6.8	2.60	86°W	0.38	H_1, H_2, H_3
11–12	$20^h20'$	$04^h45'$	8.4	1.25	7°E	0.15	I_1, I_2, I_3, J_1, J_2
12	$04^h45'$	$08^h19'$	3.6	3.25	79°W	0.91	J_3
12	$08^h45'$	$12^h00'$	3.3	2.70	2°W	0.83	K_1, K_2
12	$12^h00'$	$15^h08'$	3.1	4.00	91°E	1.28	K_3
12	$15^h50'$	$20^h08'$	4.3	3.50	35°W	0.81	L, M
12–13	$20^h08'$	$04^h45'$	8.6	3.00	45°W	0.35	N, O, P, Q, R
13	$04^h45'$	$05^h43'$	1.0	0.60	13°W	0.60	—
13	$06^h10'$	$13^h14'$	7.1	11.1	7°E	1.57	S, T, U, V
13	$15^h28'$	$20^h20'$	4.9	4.80	N	0.98	W

Time is recorded as 60th meridian; to correct to L.C.T. subtract 26 minutes. ΔD is amount of drift in nautical miles; drift direction is measured east and west of north; drift speed is given in nautical miles per hour.

[1] Oceanographic sections and horizontal projections illustrating horizontal gradients of temperature, salinity, and oxygen in this locality are given by figures 14, 15, 17, 18, and 22 to 36 in Seiwell (1934).

METHODS

Water samples were obtained by means of Nansen type reversing water bottles to each of which were attached two Richter and Wieser reversing thermometers.

Serial Observations

The total time of actual operation at station 2639 was approximately four and one quarter days; during the first three and one quarter days the entire water column from surface to bottom (5090 meters) was sampled, but, during the final twenty-four hours, sampling was limited to the upper 1200 meters. The vertical distance between samples was approximately: 0 to 100 meters = 50 meters; 100 to 1400 meters = 100 meters; 1400 to 2000 meters = 200 meters; and 2000 to 5000 meters = 250 meters. To complete a sampling of the entire water column required three separate lowerings of the water bottles and thermometers for which about six hours were required; sampling of the upper 1200 meters was completed with one lowering of water bottles and thermometers and required only about two hours. Thus, within the first three and one quarter days, eleven complete samplings of the entire water column were made, and, for the final twenty-four hours, twelve samplings of the upper 1200 meters were obtained.

Two reversing thermometers were always attached to each water bottle and in each lowering three or four of the water bottles carried both protected and unprotected thermometers, spaced at appropriate intervals. The difference in readings of unprotected and protected thermometers for the same level is used for the calculation of the sampling depths.[2]

Probable Errors of the Observations

The probable error is so defined that the chances are even whether the deviation exceeds it in absolute magnitude or is less than it. Further, the probability that the error exceeds 2.4 times the probable error is 1/10, 3.8 times the probable error is 1/100, and 4.9 times the probable error is 1/1000.

The probable errors of the various determinations carried out on board "Atlantis" during the investigation were:

Oxygen = 0.03 cc/liter
Salinity = 0.02 ‰
Temperature = 0.01° to 0.02° (depending on the particular thermometer).

Determination of Exact Time of Sampling

For purposes of this investigation it was necessary to know the times at which the water bottles and thermometers actually sampled the water mass. This was determined from information obtained on board "Atlantis" which showed that for average wire angles ranging from 5° to 35°, between surface and 500 meters, the falling velocity of the messengers ranged from about 295 to 205 meters per minute; the decrease in falling velocity of the messenger being approximately 15 meters per minute for each 5 degrees the average angle of the hydrographic wire was increased.

The sampling time results recorded in table 13 are probably not in error by more than ±3 minutes.

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[2] For a discussion of this method see: Wüst, Böhnecke and Meyer (1932).

VARIATIONS OF TEMPERATURE AND OXYGEN AT STANDARD DEPTHS DURING THE 24 HOUR PERIOD[3] BEGINNING 15^h 34′, JULY 12, 1936

50 METERS

The 50 meter depth falls in the summer thermocline where a small vertical displacement of the water column will cause correspondingly large variations of temperature (Fig. 3) because of the large negative rate of change of temperature with increasing depth.

Temperature (Fig. 2). The first evident change in the course of the temperature time curve indicates that the minimum temperature (19.93°) occurred at about 18^h 00′ which was followed by a maximum (21.94°) at around 24^h, or approximately 6 hours later. Still later the temperature dropped to 20.21° at 4^h 00′, then fluctuated within 0.15° until 10^h 00′ when it rapidly dropped to a second minimum (19.54°) at 12^h, or about 12 hours after the preceding temperature maximum.

Oxygen. Analysis of oxygen variations with time at the 50 meter level are complicated by the occurrence of an oxygen maximum at a nearby depth (Fig. 4). During July 9 to 12, oxygen variations were inverse to temperature variations but the observation intervals were too wide to show details. In the more detailed twenty-four hour period, there was a general correspondence between the occurrence of oxygen maximum values and temperature minimum values and vice versa, although a certain amount of discrepancy is to be expected from the effects of biological activity (photosynthesis and respiration) at this level. It appears from the general correspondence of oxygen maxima and temperature minima that the normal position of the oxygen maximum was at a depth somewhat below 50 meters.

That increases in oxygen content, which corresponded in general with decreases in temperature, cannot be entirely the result of photosynthesis is shown by the fact that these increases occurred between the time of sunset and sunrise (indicated by the heavy lines, Fig. 2) as well as between the time of sunrise and sunset.

100 METERS

The 100 meter level represented the approximate lower limit of the summer thermocline, and the vertical variation of temperature in the overlying water was somewhat greater than that in the water below (Fig. 3).

Temperature (Fig. 2). At 100 meters only one distinct temperature minimum and one maximum occurred in 24 hours, both of which corresponded respectively to the time of the first minimum and maximum of the 50 meter layer. The temperature minimum (18.25°) occurred at 18^h 00′ and was followed by a temperature maximum (18.91°) about six hours later at 24^h 00′. Still later the temperature decreased to 18.50° at 4^h 00′ and then fluctuated by not more than 0.09° to the end of the observational period at 15^h 34′.

Oxygen. In general the temperature oxygen relationship at the 100 meter level was opposite to that recorded for 50 meters; a rising oxygen content corresponding to a rising temperature and vice versa. Thus, corresponding with the temperature minimum at 18^h 00′ and maximum at 24^h 00′ there were agreeing peaks and valleys in the oxvgen curve.

[3] Local civil time of mean station position.

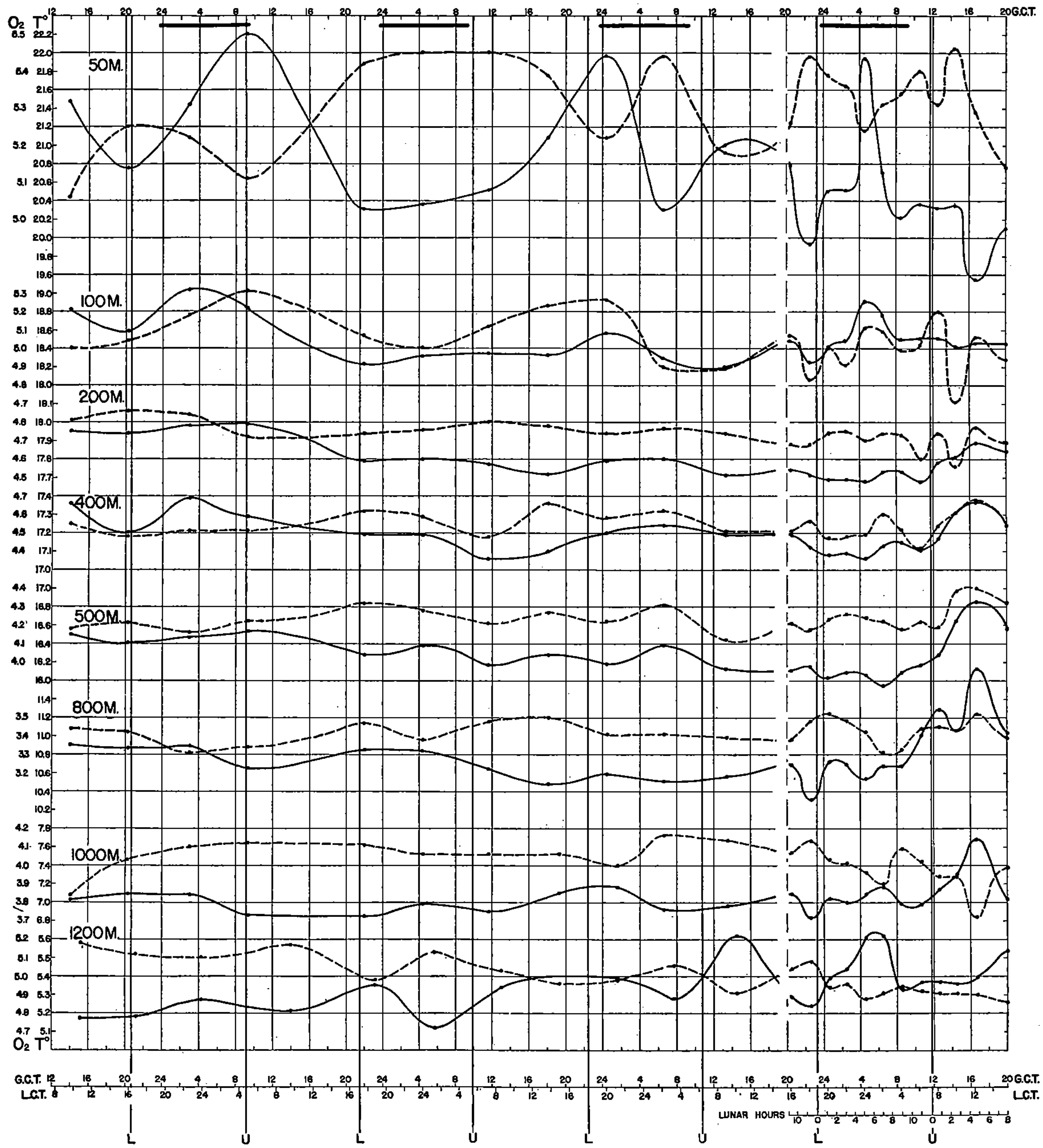

FIG. 2.—Time variation of temperature and oxygen for various depths at "Atlantis" station 2639 during 9^h 30′ July 9 to 15^h 45′ July 13, 1936 (local civil time), the final 24 hour observation period is more detailed (see text). Symbols *L* and *U* refer to times of lower and upper culminations of the moon. Temperature is indicated by full line, oxygen by broken line.

However, it is shown by the plotted data for the 100 meter level (Fig. 2) that oscillations of the oxygen curve may not agree necessarily with the temperature curve. On July 12, between 20^h and 22^h while the temperature increased from 18.41° to 18.48°, the oxygen decreased from 5.00 to 4.91 cc per liter; and between 4^h and 16^h (July 13) when the maximum temperature change was from 18.42° to 18.51°, the oxygen curve

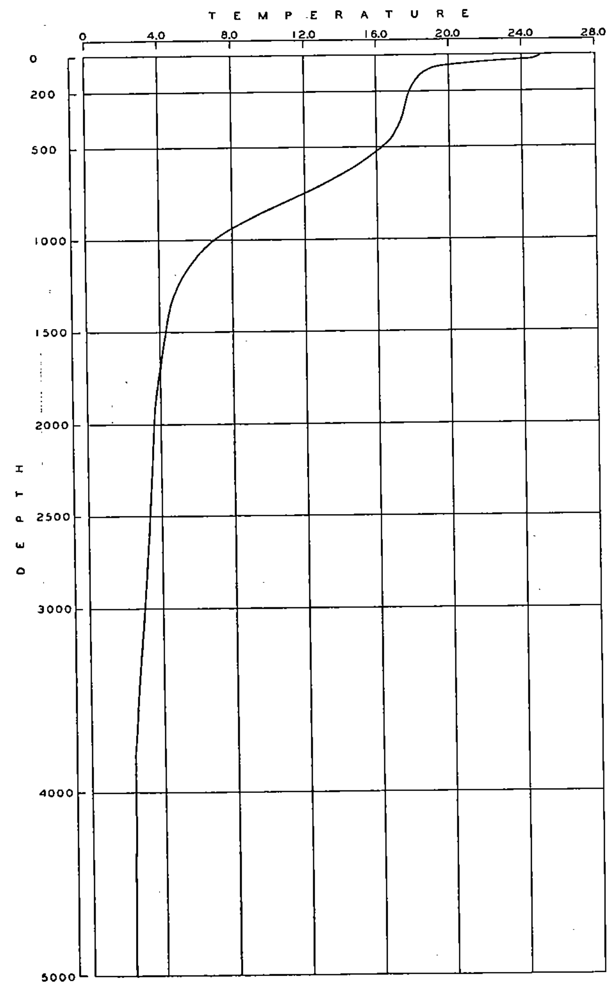

FIG. 3.—Mean vertical distribution of temperature at "Atlantis" station 2639.

passed through two complete well defined oscillations. If we may assume that vertical oscillations of the water column are indicated by temperature changes, as seems reasonable, then the oxygen nonconformities were likely the result of the biological influences at this depth.

200 METERS

The 200 meter level occurred below the summer thermocline and above the main thermocline; the water above and below was relatively homogeneous and vertical displacements produced less variation than at either the 50 or 100 meter depth.

Temperature. From a minimum value of 17.69° at $00^h\ 06'$ on July 13 the temperature increased to a not well defined maxima of 17.74° in about three hours ($3^h\ 18'$). Still later at $6^h\ 06'$ a second minimum of 17.68° was reached to be followed by a maximum of 17.89° at $12^h\ 18'$. The time between the first minima and final maxima was a little more than twelve hours.

Oxygen. In general the rise and fall of the oxygen curve was similar to that of the temperature curve. Small irregularities occurred, but the courses of both curves were similar and the principal points of agreement were well defined.

400 Meters

The 400 meter depth marked the upper part of the main thermocline; the oxygen and temperature gradients being greatest in the underlying water.

Temperature. The first, but not well defined, temperature minimum (17.06°) occurred

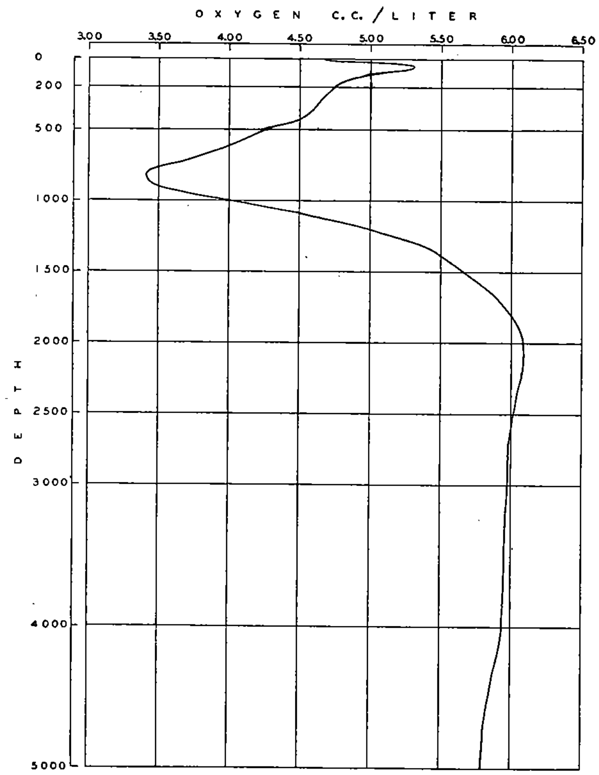

Fig. 4.—Mean vertical distribution of oxygen at "Atlantis" station 2639.

at $00^h\ 06'$ (July 13) and was followed by a maximum (17.15°) a little more than three hours later at $3^h\ 18'$. A little less than three hours later a second minimum (17.11°) occurred at $6^h\ 06'$ and was followed by a well defined maximum (17.37°) at $12^h\ 06'$, or about six hours later. The approximate time from the first minimum to the final maximum was twelve hours.

Oxygen. Throughout the entire observational period there was a close agreement of oxygen and temperature variations.

500 Meters

The 500 meter level laid within the principal thermocline.

Temperature. The first depression in the temperature time curve (16.03°) occurred at $19^h\ 30'$ (July 12) and was followed by a peak (16.10°) at $22^h\ 30'$ and then a second depression (15.95°) at $1^h\ 42'$ (July 13) and a second peak (16.86°) at $12^h\ 42'$. The intervals between peaks and valleys were approximately 3 and 11 hours, suggesting the presence of 6 and 24 hour periods.

Oxygen. The variations of oxygen and temperature were in agreement; small discrepancies occurred but their magnitude was not sufficient to alter the main trend.

800 Meters

The 800 meter level occurred in the principal thermocline.

Temperature. The first minimum (10.31°) was found at $18^h\ 00'$ (July 12) and was followed by a maximum (10.75°) about three hours later at $21^h\ 00'$. Three hours later a second minimum (10.53°) had occurred at $00^h\ 00'$ and twelve and one half hours later a second maximum (11.75°) at $12^h\ 30'$ (July 13).

Oxygen. Variations in the oxygen curve are complicated since the normal depth o the minimum oxygen content at this station was at about 850 meters (Fig. 4), and there was no particular correspondence between the trajectories of oxygen and temperature values.

1000 Meters

The 1000 meter level laid in the lower part of the principal thermocline.

Temperature. The course of the temperature curve indicates a minimum value (6.84°) at $18^h\ 30'$ (July 12) and about three and one half hours later a second minimum (7.00°) at $22^h\ 00'$ with a maximum about midway between. Still later the curve passed through a second maximum (7.17°) at about $2^h\ 00'$ (July 13), followed by the third minimum (6.96°) three and one half hours later at $5^h\ 30'$ and the third maximum (7.69°) at $12^h\ 00'$.

Oxygen. The direction of the oxygen gradient at 1000 meters was such that vertical movements of the water column will cause the oxygen curve to vary in the opposite direction to temperature (Fig. 4). Thus, a comparison of results above and below the oxygen minimum concentration suggests strongly that the observed variations of oceanographic factors are caused primarily by vertical displacements of the water column.

1200 Meters

The 1200 meter level was very near the lower limit of the principal thermocline.

Temperature. The temperature rose from its first minimum value (5.24°) at $18^h\ 10'$ (July 12) to a maximum (5.64°) at $1^h\ 00'$ (7 hour interval); it then decreased to a second minimum (5.33°) at $4^h\ 24'$ and finally was approaching a second maximum value ($>5.54°$) when observations ended at $15^h\ 44'$.

Oxygen. Throughout the observational period oxygen increases corresponded to temperature decreases and vice versa as in the case of the 1000 meter level.

A summary of the times of occurrence of maxima and minima values of temperature at various fixed levels during the final 24 hour observation period is given in table 2. It is seen that the time intervals between minima and maxima, or vice versa, divide themselves into three distinct classes with values of 2.8 to 3.5, 6 to 6.8 and 11 to 12.5 solar hours, the latter two of which may correspond to periods of 12 and 24 lunar hours and be connected with the tides.

TABLE 2

Meters Depth	Minimum Value	Minimum Time	Minimum Cul.	Approx. Interval	$\frac{dT^\circ}{dt}$	Maximum Value	Maximum Time	Maximum Cul.	Approx. Interval	$\frac{dT^\circ}{dt}$	Minimum Value	Minimum Time	Minimum Cul.	Approx. Interval	$\frac{dT^\circ}{dt}$	Maximum Value	Maximum Time	Maximum Cul.	Approx. Interval
50	19.93°	$18^h\ 00'$	$L-0.9^h$	6^h	0.335	21.94°	$00^h\ 00'$	$L+5.1^h$	12^h	0.20	19.54°	12^h	$U+4.7^h$						
100	18.25°	$18^h\ 00'$	$L-0.9^h$	6^h	0.11	18.91°	$00^h\ 00'$	$L+5.1^h$											
200	17.69°	$00^h\ 06'$	$L+5.2^h$	3.2^h	0.016	17.74°	$3^h\ 18'$	$L+8.4^h$	2.8^h	0.021	17.68°	$6^h\ 06'$	$L+11.2^h$	6.2^h	0.034	17.89°	$12^h\ 18'$	$U+5^h$	
400	17.06°	$00^h\ 06'$	$L+5.2^h$	3.2^h	0.028	17.15°	$3^h\ 18'$	$L+8.4^h$	2.8^h	0.014	17.11°	$6^h\ 06'$	$L+11.2^h$	6^h	0.043	17.37°	$12^h\ 06'$	$U+4.8^h$	
500	16.03°	$19^h\ 30'$	$L+0.6^h$	3^h	0.023	16.10°	$22^h\ 30'$	$L+3.6^h$	3.2^h	0.047	15.95°	$1^h\ 42'$	$L+6.8^h$	11^h	0.083	16.86°	$12^h\ 42'$	$U+5.4^h$	
800	10.31°	$18^h\ 00'$	$L-0.9^h$	3^h	0.147	10.75°	$21^h\ 00'$	$L+2.1^h$	3^h	0.073	10.53°	$00^h\ 00'$	$L+5.1^h$	12.5^h	0.10	11.75°	$12^h\ 30'$	$U+5.2^h$	
1000	6.84°	$18^h\ 30'$	$L-0.4^h$	—	—	—	—	—	3.5^h	—	7.00°	$22^h\ 00'$	$L+3.1^h$	4^h	0.043	7.17°	$2^h\ 00'$	$L+7.1^h$	3.5^h
1000	6.96°	$5^h\ 30'$	$U-1.8^h$	6.5^h	0.112	7.69°	$12^h\ 00'$	$U+4.7^h$											
1200	5.24°	$18^h\ 10'$	$L-0.7^h$	6.8^h	0.058	5.64°	$1^h\ 00'$	$L+6.1^h$	3.4^h	0.091	5.33°	$4^h\ 24'$	$L+9.5^h$	11.3^h+		5.54°+	$15^h\ 44'+$		

Scaled values for maxima and minima of temperature curves for various depths during 24 hour period: July 12, $15^h\ 34'$ to July 13, $15^h\ 34'$. The column headed Cul. gives the number of hours before or after the culminations of the moon (L and U). The occurrence of temperature maxima and minima with respect to the moon's culminations vary from depth to depth, but frequently there was a temperature minimum within one hour before the moon's lower culmination, on July 12, and a maximum 4 to 6 hours afterwards; similarly a temperature maximum frequently occurred about 5 hours after the moon's upper culmination. The rate of change of temperature with time is indicated approximately by dT^0/dt, degrees per hour. This value in part depends on the rate of change of temperature with depth and is of significant magnitude.

The times of the occurrence of maxima and minima temperature values with respect to the moon's culmination for the 24 hour period beginning $15^h\ 34'$, July 12, are indicated in table 2, and, while these vary from depth to depth, it is seen for most of the depths examined that there was a temperature minimum within one hour before the lower culmination of the moon on July 12 and a maximum about 4 to 6 hours afterwards. Also, a temperature maximum frequently occurred about 5 hours after the upper culmination of the moon.

Temperature variations at standard depths with respect to the culminations of the moon during the entire observational period may be obtained from figure 2, but, with the exception of the final 24 hours, the relationship has little meaning because the observations were too far apart (approximately 6 hours). Thus, considering temperatures at the time of the culminations of the moon at $18^h\ 52'$ on July 12 (lower) and at $7^h\ 20'$ on July 13 (upper, L.C.T.) there were, for all depths examined, higher temperatures at the time of the moon's upper culmination.

The rate of change of temperature with time between 50 and 1200 meters is indicated approximately by dT°/dt in table 2. The rate of change of temperature with time will be determined largely by the rate of change of temperature with depth; even at 1200 meters it may be relatively large, e.g., $dT^\circ/dt=0.091$. The maximum dT°/dt value recorded was 0.335° per hour at 50 meters depth.

DEPTH VARIATIONS OF ISOTHERMS

Table 3 summarizes the variation in depth of selected isotherms between mean depths of 60 and 4067 meters as observed during the 102 hour period at station 2639; the variations may be used as indicators of vertical movements of the water column (page 17). Below 100 meters the vertical displacement of the isotherms ranged from 52 to 185 meters (the error of estimating the amount of displacement increases for great depths where the temperature gradient is very small), and during a single twenty-four hour period the isotherms between 60 and 1265 meters underwent vertical displacements of 34 to 74 meters.

TABLE 3

Isotherm	Mean Depth	Total Depth Range $9^h30'$, 7/9 to $15^h34'$, 7/13		Total Displacement	Total Depth Range $15^h34'$, 7/12 to $15^h34'$, 7/13		Total Displacement
20.0°	60	42	82	40	42	76	34
18.5°	101	83	135	52	83	118	35
17.0°	428	408	482	74	408	482	74
15.0°	591	560	633	73	571	633	62
10.0°	816	777	841	64	777	830	53
5.0°	1265	1240	1305	65	1242	1285	43
4.0°	1675	1647	1728	81			
3.0°	3000	2958	3036	78			
2.4°	4067	3973	4158	185			

Vertical range of selected isotherms between 60 and 4067 meters observed at Station 2639.

The approximate times at which the highest and lowest vertical positions of various isotherms (20.0°, 18.5°, 17.0°, 15.0°, 10.0°) were reached during the 24 hour observational period, beginning $15^h\ 30'$ on July 12, are given in table 4, and possible connections with the tidal wave are suggested by the approximate 6 and 12 hour intervals (Fig. 5). The data also indicate the presence of other unknown disturbances which appear to play a significant part in these variations.

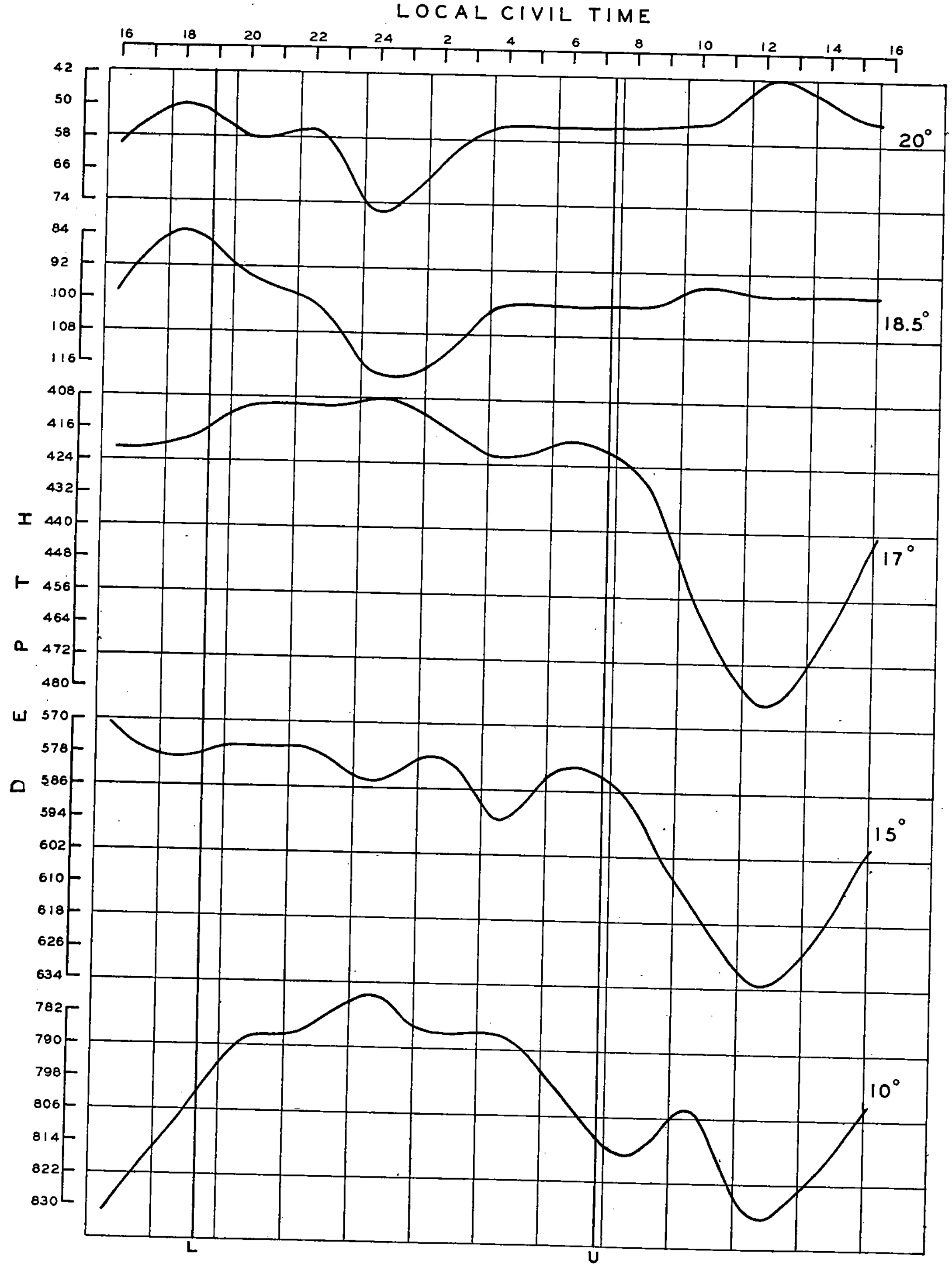

FIG. 5.—Time depth variation of 20°, 18.5°, 17°, 15°, and 10° isotherms at "Atlantis" station 2639 for the twenty-four hour period beginning $15^h\ 30'$ July 12, 1936 (see text). Symbols *L* and *U* refer to times of lower and upper culminations of moon.

TABLE 4

Isotherm	High Depth	High Time	Interval	Low Depth	Low Time	Interval	High Depth	High Time	Interval	Low Depth	Low Time	Interval
20.0°	50	$18^h00'$	6^h	76	$24^h00'$	$12^h24'$	42	$12^h24'$	—	—	—	—
18.5°	83	$18^h00'$	$6^h12'$	118	$00^h12'$	—	—	—	—	—	—	—
17.0°	408	$00^h12'$	$3^h48'$	422	$4^h00'$	$2^h15'$	418	$6^h15'$	$6^h05'$	482	$12^h20'$	—
15.0°	—	—	—	579	$18^h00'$	—	—	—	$6^h10'$	584	$00^h10'$	$1^h50'$
	578	$02^h00'$	$2^h08'$	593	$4^h08'$	$2^h10'$	580	$6^h18'$	$6^h07'$	633	$12^h25'$	—
10.0°	777	$00^h10'$	$8^h00'$	815	$8^h10'$	$2^h00'$	803	$10^h10'$	$2^h20'$	830	$12^h30'$	—

Depths (meters) and times (L.C.T.) at which high and low positions of various isotherms were reached during the twenty four hour period beginning July 12, 1936, $15^h34'$. Local civil time refers to the mean position of station 2639 (66°25′W, 37°07′N).

CAUSE OF VERTICAL DISPLACEMENTS OF ISOTHERMS

Before proceeding to a quantitative discussion of the vertical oscillations of the water column, evidence supporting the contention that vertical oscillations of the isotherms at station 2639 are primarily the result of vertical movements of the water column and not of horizontal shifts will be considered by studying the horizontal gradients of the 17° and 10° isotherms in the vicinity of station 2639.

Long Period Variations in Depths of 17° and 10° Isotherms in the Vicinity of Station 2639

Temperature data from 33 "Atlantis" stations, within a radius of about 200 miles from station 2639 (Fig. 1), were examined and the depths of the 17° and 10° isotherms scaled from temperature depth curves. The observations at these stations were taken between November 24, 1931 and July 28, 1933; the depth variations of the 10° and 17° isotherms are summarized in table 5.

TABLE 5

	1931 Nov. 24–26	1931 Dec. 6–7	1932 Feb. 14–17	1932 Apr. 18–20	1932 Aug. 18–31	1932 Dec. 2–4	1933 Feb. 12–13	Totals
10° range	810–916	888–1000	885–1025	840–930	825–907	818–915	863–935	810–1025
10° mean	878	944	974	902	872	872	899	903
$\Delta Z_{10°}$	106	112	140	90	82	97	72	215
17° range	476–535	510–600	510–605	475–600	475–535	452–545	478–605	452–605
17° mean	507	560	571	528	496	503	542	525
$\Delta Z_{17°}$	59	90	95	125	60	93	127	153
No. of stations	4	4	5	4	9	5	2	33

Depth range of 10° and 17° isotherms at "Atlantis" stations within a 200 mile radius of station 2639. ΔZ = vertical range of isotherm during indicated time interval.

The mean depth of the 10° isotherm at station 2639 during July 9 to 13, 1936 was calculated to be 816 meters and its vertical range during this period 64 meters (page 13). However, table 5 shows that from November 24, 1931 to February 13, 1933 the mean depth of this isotherm was 903 meters and its total vertical range 215 meters. Hence, during this 15 month interval the 10° isotherm was subjected to influences which greatly outweigh the effects of daily and half-daily tides. Table 5 illustrates certain trends, significant of which are, that between November 1931 and February 1933 the 10° isotherm reached its greatest mean depth near the end of winter, and its least mean depth in mid summer, or at the same season that station 2639 was made. This seasonal difference, in part, explains the high position of the 10° isotherm in the water column at station 2639.

The amount of vertical displacement of the 10° isotherm also appears to show a seasonal effect; table 5 illustrating that the maximum observed displacement occurred near the end of winter, coinciding with the maximum mean depth of this isotherm and a significantly smaller displacement occurred in summer when the mean depth of the isotherm was least.

The mean depth of the 17° isotherm at station 2639 during July 9 to 13, 1936 was calculated to be 428 meters, and its maximum vertical range 74 meters (page 13). The data in table 5 show that the greatest mean depth (571 meters) occurred near the end of winter and the least mean depth (496 meters) near mid summer; and that the maximum observed vertical displacement occurred in winter and the minimum in mid summer.

Thus, the 10° and 17° isotherms in the vicinity of station 2639 are similar in their seasonal changes; in winter both isotherms appear to occupy their greatest mean depth in the water column and go through their maximum vertical displacement; whereas in summer they both occupy higher mean positions in the water column and exhibit less vertical displacement. The fact that station 2639 was made in mid summer may account for the relatively high mean depths of the 10° and 17° isotherms. However, since table 5 shows a not too good agreement between the December 1931 and February 1932 values and the December 1932 and February 1933 values there is the suggestion of an annual variation superimposed on the seasonal variation of the mean depth of these isotherms which is no doubt the result of annual changes in the circulation. Thus, a seasonal variation is defined as:

$$f(x+\alpha) = f(x)$$

where α is the period. The fact that values of the mean depth do not recur after a fixed interval is due partly to the effect of diurnal and semidiurnal variations as well as probably the effect of an annual variation. The incompleteness of the data, however, prevent the specification of the magnitude of seasonal and annual variation.

Determination of Maximum Horizontal Gradients of 17° and 10° Isotherms in Vicinity of Station 2639

Since the possibility exists that the vertical oscillations of the isotherms at station 2639 to some extent may be caused by periodic horizontal displacements of the water mass (accompanying the tidal wave) it seemed advisable to examine the horizontal gradients of various isotherms in the vicinity of this station. The foregoing discussion of the short and long period variations of isothermal depths illustrates the difficulty of determining horizontal gradients of isotherms (or any other characteristic for a particular time) in the vicinity of station 2639. Consequently, for purposes of argument, the maximum horizontal gradient which might possibly exist in the vicinity of this station at any time (from the data at hand) has been calculated and the magnitude of the required horizontal displacement to produce the observed vertical displacements of the isotherms has been determined therefrom. This treatment should supply information regarding the reasonableness of suggesting that vertical displacements of the isotherms in the vicinity of station 2639 were caused by horizontal displacements of the water layers which may be associated with the tidal wave.

The maximum horizontal gradient of the 10° and 17° isotherms have been determined on the basis of the assumption that the depth variations of the 10° and 17° isotherms observed between November 24, 1931 and July 28, 1933 (table 5) could exist at any time in the vicinity of station 2639 (this, being no doubt an impossibility, serves for argumentative purposes). The following results were obtained:

1. Maximum gradient of 10° isotherm: 1.33 meters/kilometer
2. Maximum gradient of 17° isotherm: 1.00 meters/kilometer

The Horizontal Displacements Required to Produce the Observed Vertical Displacements of the 10° and 17° Isotherms

10° isotherm. From the time depth curve of this isotherm (Fig. 5) the following data for the 24 hour period of July 12–13 are obtained:

highest level: 777 meters, occurred about $00^h\ 00'$ (L.C.T.)
lowest level: 830 meters, occurred about $12^h\ 30'$ (L.C.T.)

The average rate of change of elevation during this period was 4.24 meters per hour. To produce this same effect by a pure horizontal displacement on the basis of a maximum horizontal gradient of 1.33 meters/kilometer, a mean horizontal velocity of the water particles of 3.2 kilometers per hour or 89 centimeters per second would have been required. Since the normal horizontal gradient is probably only one half to one third of the amount used in the above calculation, the improbable horizontal velocities of around 200 centimeters per second would have been required to produce the observed vertical displacements of this isotherm.

17° isotherm. From the time depth curve of this isotherm (Fig. 5) the following data for the 24 hour period of July 12–13 are obtained:

highest level: 408 meters, occurred about $00^h\ 00'$ (L.C.T.)
lowest level: 482 meters, occurred about $12^h\ 30'$ (L.C.T.)

The average rate of change of elevation during this period was 5.92 meters per hour. To produce this same effect by a pure horizontal displacement on the basis of a maximum horizontal gradient of 1.00 meters per kilometer, a mean horizontal velocity of the water particles of 5.9 kilometers per hour or 164 centimeters per second would have been required. Since the normal horizontal gradient is probably one half to one third of the amount used in this calculation, the absurd horizontal velocities of about 350 to 450 centimeters per second would have been required to produce the observed vertical displacements of this isotherm.

Thus, on the basis of the foregoing discussion it is reasonable to conclude that vertical oscillations of temperature (and other properties of sea water) in the locality represented by station 2639 resulted primarily from vertical oscillations of the water column rather than from horizontal displacements of the water layers.

ANALYSIS OF TIME VARIATIONS IN DEPTH OF ISOTHERMS DURING THE 24 HOUR PERIOD

Method of Harmonic Analysis

Since the preceding discussion suggests a connection between vertical oscillations of the isotherms and the tides the resulting amplitudes and phases of the oscillations may be determined by fitting a Fourier series with a limited number of terms to the observed data. This means that we shall first decompose the isotherm time depth curves (Fig. 5) into simpler waves having periods the same as the tidal wave (24 and 12 lunar hours). The depths of the various isotherms plotted as a function of time individually specify graphically the functions for which we wish to find Fourier series, each of which will represent, with a limited number of terms, as closely as possible the individual graphs of the observations. The general method used in this investigation for obtaining Fourier coefficients is outlined briefly in the following discussion.

From the given graph (Fig. 5) the values of the function, $u(x)$, corresponding to the values,

$$0, \quad \frac{2\pi}{n}, \cdots \frac{2(n-1)\pi}{n} \text{ of } x$$

are obtained. And, it is desired to find a sum:

$$\begin{aligned} f(x) &= a_0 + a_1 \cos x + a_2 \cos 2x + \cdots a_r \cos rx \\ &\quad + b_1 \sin x + b_2 \sin 2x + \cdots b_r \sin rx \end{aligned}$$

which furnishes the best possible representation of the function $u(x)$ (where $2r+1 \leqq n$). Denoting $u(0)$ by u_0, $u(2\pi/n)$ by u_1 etc. and substituting in the above equation the following equations of condition for the determination of the coefficients $a_0, a_1, \cdots a_r; b_1, b_2, \cdots b_r$ are obtained:

$$\begin{aligned} u_0 &= a_0 + a_1 + a_2 + \cdots a_r \\ u_1 &= a_0 + \sum_{k=1}^{k=r} a_k \cos \frac{2k\pi}{n} + \sum_{k=1}^{k=r} b_k \sin \frac{2k\pi}{n} \\ &\vdots \\ u_p &= a_0 + \sum_{k=1}^{k=r} a_k \cos \frac{2kp\pi}{n} + \sum_{k=1}^{k=r} b_k \sin \frac{2kp\pi}{n} \\ &\vdots \\ u_{n-1} &= a_0 + \sum_{k=1}^{k=r} a_k \cos \frac{2k(n-1)\pi}{n} + \sum_{k=1}^{k=r} b_k \sin \frac{2k(n-1)\pi}{n}. \end{aligned}$$

For the case where there are more equations than unknowns the method of least squares is now applied to find the best possible solution of these equations; and as a final result the normal equations for the coefficients $a_0, a_1, \cdots a_r$ and $b_1, b_2, \cdots b_r$ are obtained:

$$\begin{aligned} na_0 &= u_0 + u_1 + u_2 + \cdots u_{n-1} \\ \frac{n}{2} a_r &= u_0 + u_1 \cos \frac{2r\pi}{n} + u_2 \cos \frac{4r\pi}{n} + \cdots + u_{n-1} \cos \frac{2(n-1)\pi}{n} \\ \frac{n}{2} b_r &= u_1 \sin \frac{2r\pi}{n} + u_2 \sin \frac{4r\pi}{n} + \cdots + u_{n-1} \sin \frac{2(n-1)\pi}{n}. \end{aligned}$$

These equations give the coefficients $a_0, a_1, \cdots a_r; b_1, \cdots b_r$ which enable the arbitrary function, $f(x)$, to be expressed as nearly as possible as the sum of a limited number of sine and cosine terms.[4]

[4] For a more detailed discussion of this method the reader is referred to Whittaker and Robinson (1929).

As a final step the resulting equation is put in the form:

$$c_0 + c_1 \cos (x - \alpha_1) + c_2 \cos (2x - \alpha_2)$$

the amplitudes, c, and the phases, α, are connected with the a's and b's by the relations:

$$\alpha = \cos^{-1} \frac{a}{b}$$

$$c = \sqrt{a^2 + b^2}$$

This analysis is applied to the 20° (42 to 63 meters), 17° (408 to 481 meters) and 10° (777 to 830 meters) isotherms. The 20° isotherm lies within the summer or temporary thermocline, the 17° isotherm marks approximately the beginning of the main thermocline, and the 10° isotherm lies near the mid point of the main thermocline (Fig. 3).

The data for carrying out the analysis were obtained during the 24 hour period which began at about $15^h\ 30'$ (L.C.T.) on July 12, 1936; the observations were separated by approximate two hour intervals. At the mean position of the station (page 5) the lower culmination of the moon occurred at $18^h\ 52'$ on July 12 and the upper culmination at $7^h\ 20'$ on July 13. Beginning with the time of the moon's lower culmination, the entire period was divided into twenty-four lunar hours (12.46 solar hours = 12 lunar hours) and the depths of the isotherms concerned were scaled for each lunar hour (Fig. 5). The lunar time beginning from the moon's lower culmination is designated as t. However, since the observations did not extend for a full twenty-four lunar hours beyond the time of the moon's lower culmination on July 12, data for three lunar hours previous to this time were used. This reference time (which begins three hours before the moon's lower culmination on July 12) is designated as t'[5] and was used for the harmonic analysis. The final results for the phase determinations are represented as the number of lunar hours after the moon's lower culmination; the translation depending on the simple relation:

$$t = t' + 3.$$

Results of Harmonic Analysis

20° isotherm. Harmonic analysis gives for variations in depth of this isotherm:

$$H_{20^\circ} = 55.4 + 7.44 \cos \frac{2\pi}{24} (t - 5.2^h) + 1.81 \cos \frac{2\pi}{12} (t - 5.8^h) \qquad (1)$$

where, t, represents the number of lunar hours after the lower culmination of the moon.

The amplitude of the 24 hour wave is about four times that of the 12 hour wave. Since the phases of both waves are approximately the same, the maximum downward displacement of the water column at a depth of 55 meters should occur about five and one half lunar hours after the lower culmination of the moon.

The residue, R_{20°, is defined:

$$R_{20^\circ} = O_{20^\circ} - H_{20^\circ} \qquad (2)$$

where O_{20° is the observed depth of the 20° isotherm (Fig. 5) and H_{20° the depth as calculated from equation 1; it is graphically illustrated in figure 6, and the points represent

[5] Thus, when $t = 0^h.$, $t' = -3^h$.

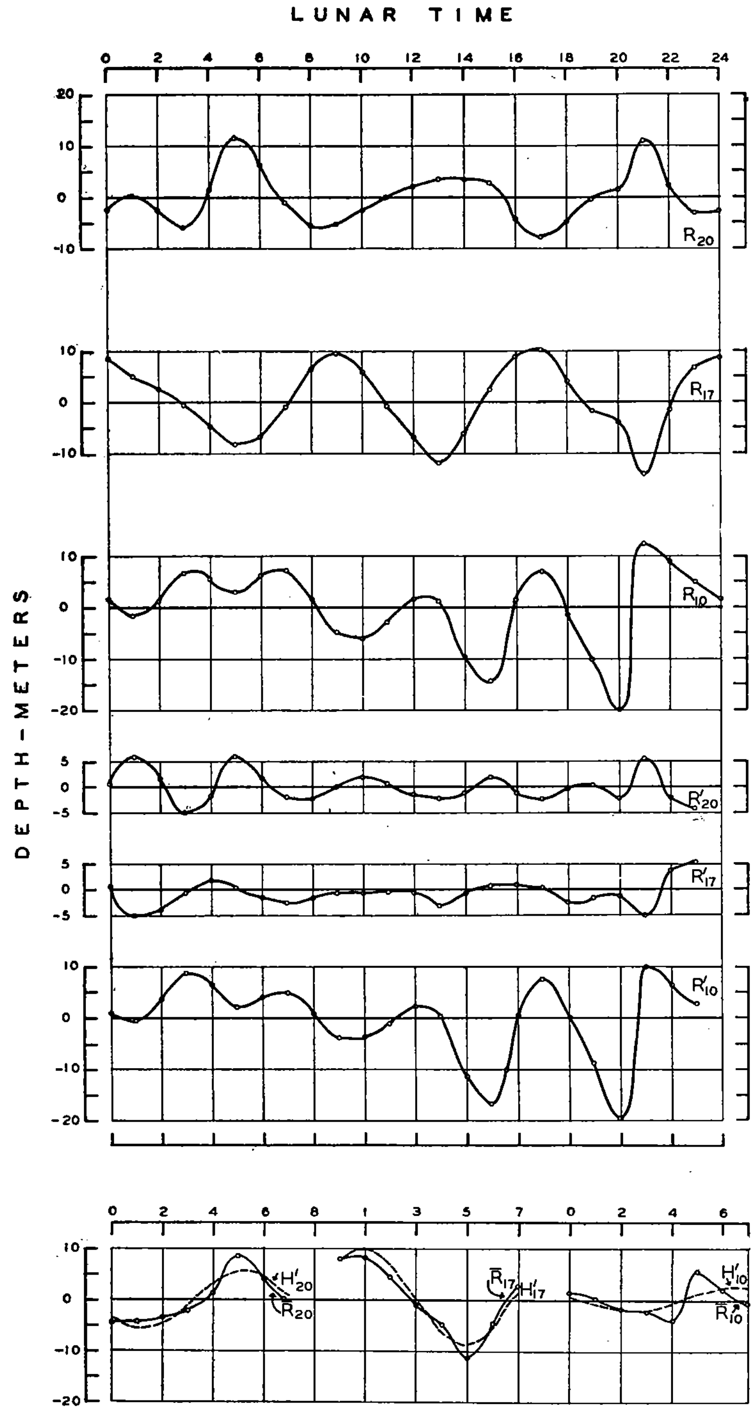

FIG. 6.—First and second residues for 20°, 17°, and 10; isotherms; lower three curves represent the average first residue of the eight hour intervals illustrating the convergence of the observational data with the theoretical equation. For explanations see text. Scale of lunar time begins with the moon's lower culmination on July 12, 1936.

an average deviation[6] of 3.84 meters for $H_{20°}$ values. The curve (Fig. 6) appears to possess a definite periodicity, the time between the occurrence of two successive maxima or two successive minima amounting to about eight hours. Averaging the three 8 lunar hour periods, the results headed $\bar{R}_{20°}$ in table 6 are obtained (Fig. 6).

TABLE 6

t (lunar hours)	$\bar{R}_{20°}$ (meters)	$H'_{20°}$ (meters)	$\bar{R}'_{20°}$ (meters)
0^h	−4.2	−3.3	−0.9
1^h	−4.2	−5.5	+1.3
2^h	−3.3	−4.5	+1.2
3^h	−2.1	−0.9	−1.2
4^h	+1.6	+3.3	−1.7
5^h	+8.6	+5.5	+3.1
6^h	+4.1	+4.5	−0.4
7^h	−0.4	+0.9	−1.3

Harmonic analysis of these data gives:

$$H'_{20°} = 5.6 \cos \frac{2\pi}{8}(t-5.2^h) \tag{3}$$

and the results calculated for each lunar hour (0 to 7) are given in the column headed H'_{20^0} in table 6. The final average residue ($\bar{R}'_{20°}$) defined as follows:

$$\bar{R}'_{20°} = \bar{R}_{20°} - H'_{20°} \tag{4}$$

is found in table 6 and illustrates the amount of convergence which can be obtained by the use of equations 1 and 3. No tidal significance can be attached to the 8 hour period derived from the first residue (equation 3). Nevertheless, for the 24 hour period beginning $15^h\ 30'$ (L.C.T.) on July 12, 1936, a good convergence is obtained by combining the 24 and 12 hour harmonics (apparently associated with the tidal wave) with an 8 hour harmonic so that a close approximation of the vertical oscillation of the water column at an average depth of 55 meters is given by:

$$H_{20°} = 55.4 + 7.44 \cos\frac{2\pi}{24}(t-5.2^h) + 1.81 \cos\frac{2\pi}{12}(t-5.8^h) + 5.6 \cos\frac{2\pi}{8}(t-5.2^h) \tag{5}$$

where t is the time in lunar hours after the lower culmination of the moon.

It is interesting to note that the 8 hour harmonic has an amplitude more than 3 times that of the 12 hour harmonic and about 0.75 of the 24 hour harmonic. Thus, it accounts for about 38 per cent of the vertical displacement of the water column. All three waves have approximately the same phase angle so that the maximum downward displacement of the water particles at the depth of the 20° isotherm occurred a little more than five hours after the moon's lower culmination. The final residue, $R'_{20°}$, (Fig. 6) represents an average deviation of only 2.2 meters between actual observed values for fluctuations in height of the 20° isotherm and the values as calculated by equation 5. This residue is probably in part the result of random motion of the water particles and in part an accumulation of errors; but further analysis of the data does not appear justifiable.

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[6] Average deviation: $\text{A.D.} = \dfrac{\sum_{i=1}^{n} f_i |X_i - A|}{N}$

where $A = \dfrac{\sum_{i=1}^{n} f_i X_i}{N}$.

17° isotherm. Harmonic analysis gives for variations in the depth of this isotherm:

$$H_{17^\circ} = 429.7 + 27.5 \cos \frac{2\pi}{24}(t-16.6^h) + 14.0 \cos \frac{2\pi}{12}(t-5.2^h) \tag{6}$$

where t is the number of lunar hours after the moon's lower culmination.

The amplitude of the 24 hour wave is twice that of the 12 hour wave. The phases of both waves are such that they will reinforce each other at about 17 hours after the moon's lower culmination to produce the maximum vertical displacement of the water particles at the mean depth of this isotherm.

The residue, R_{17°, is defined (see page 19):

$$R_{17^\circ} = O_{17^\circ} - H_{17^\circ} \tag{7}$$

where O_{17° is the observed depth of the 17° isotherm and H_{17° the depth calculated from equation 6; it is graphically illustrated in figure 6 where the points represent an average deviation of 5.76 meters between observed and H_{17° values. Since the curve shows a well defined 8 hour period, also characteristic of the 20° residue (page 21), it was treated similarly by averaging the three 8 lunar hour periods and subjecting to harmonic analysis. The resulting average residue, $\bar{R}_{17^\circ}$, is given in table 7 (Fig. 6).

TABLE 7

t (lunar hours)	$\bar{R}_{17^\circ}$ (meters)	H'_{17° (meters)	$\bar{R}'_{17^\circ}$ (meters)
0	+7.9	+7.8	+0.1
1	+8.3	+10.1	−1.8
2	+4.4	+6.7	−2.3
3	−1.0	0.0	−1.0
4	−4.9	−6.4	+1.5
5	−11.3	−8.7	−2.6
6	−4.8	−5.4	+0.6
7	+2.7	+1.4	+1.3

Harmonic analysis of the average residue, $\bar{R}_{17^\circ}$, gives:

$$H'_{17^\circ} = 0.7 + 9.4 \cos \frac{2\pi}{8}(t-0.9^h) \tag{8}$$

and the results calculated for each lunar hour (0^h to 7^h) are given in the column headed H'_{17° in table 7.

The final residue ($\bar{R}'_{17^\circ}$) shows a well defined convergence with observed values. Combining all the harmonics an expression is obtained which gives very closely the oscillation of the 17° isotherm as observed during a single 24 hour period. Thus:

$$H_{17.0^\circ} = 430.4 + 27.5 \cos \frac{2\pi}{24}(t - 16.6^h) + 14.0 \cos \frac{2\pi}{12}(t - 5.15^h) + 9.4 \cos \frac{2\pi}{8}(t - 0.9^h) \tag{9}$$

where t is the time in lunar hours after the moon's lower culmination.

The amplitude of the 24 hour wave is twice as great as that of the 12 hour wave and the 8 hour wave is relatively the least significant of the three although of somewhat greater amplitude than that obtained from the 20° isotherm. The final average residue, $\bar{R}'_{17^\circ}$, (Fig. 6) represents an average deviation of only 1.8 meters between observed and theoretical values (from equation 9).

10° isotherm. Harmonic analysis of the observational data (Fig. 5) gives for variation in depth of this isotherm:

$$(10) \qquad H_{10^\circ} = 802.3 + 24.7 \cos \frac{2\pi}{24}(t - 17.4^h) + 4.05 \cos \frac{2\pi}{12}(t + 1.6^h)$$

where t stands for the number of lunar hours after the moon's lower culmination.

As usual, the amplitude of the 24 hour wave is the greater, being more than six times that of the 12 hour wave. The two waves interfere in such a way that the maximum displacement occurs 2 to 3 hours before the moon's lower culmination.

The residue, R_{10°, is defined as previously (pages 19 and 22):

$$(11) \qquad R_{10^\circ} = O_{10^\circ} - H_{10^\circ}$$

where O_{10° is the observed depth of the 10° isotherm and H_{10° the depth calculated from equation 10 (Fig. 6); showing an average deviation of 5.9 meters between the observed and theoretical values. The curve of this residue when plotted as a function of time does not show a well defined eight lunar hour period as in the case of the 20° and 10° isotherms (pages 21 and 22); instead, the time from maximum to maximum or minimum to minimum is about five hours in most cases. Nevertheless, this first residue, R_{10°, has been averaged for three eight hour intervals (entered in table 8 as $\overline{R}_{10^\circ}$) and analyzed for an 8 hour period. The harmonic analysis of the first average residue, $\overline{R}_{10^\circ}$, gives:

$$(12) \qquad H'_{10^\circ} = 0.1 + 2.3 \cos \frac{2\pi}{8}(t - 6.4^h)$$

TABLE 8

t (lunar hours)	$\overline{R}_{10^\circ}$ (meters)	H'_{10° (meters)	$\overline{R}'_{10^\circ}$ (meters)
0	+1.6	+0.8	+0.8
1	+0.2	−0.9	+1.1
2	−1.8	−2.1	+0.3
3	−2.2	−2.0	−0.2
4	−4.1	−0.6	−3.5
5	+5.6	+1.1	+4.5
6	+1.9	+2.3	−0.4
7	−0.7	+2.2	−2.9

The second average residue, R'_{10° ($\overline{R}_{10^\circ} - H'_{10^\circ}$), shows the convergence to be less than for the 20° and 17° isotherms (pages 21 and 22). When not averaged, this residue (R'_{10°) suggests the presence of a wave having a period of approximately 5 hours (Fig. 6). The average of four 5 hour intervals, $0^h - 19^h$, (table 9) indicates an approximate amplitude of four meters, indicating it to be at least as significant as the eight hour wave.

TABLE 9

t (lunar hours)	0	1	2	3	4
Average R'_{10° (meters)	−4.3	+0.8	+4.7	+2.8	−4.4

However, within reasonable limits, the vertical oscillations of the 10° isotherm during the twenty-four hour period under consideration may be expressed by:

$$(13) \quad H_{10^\circ} = 802.4 + 24.7 \cos \frac{2\pi}{24}(t - 17.4^h) + 4.05 \cos \frac{2\pi}{12}(t + 1.6^h) + 2.3 \cos \frac{2\pi}{8}(t - 6.4^h)$$

although the agreement of calculated and observed values for the 10° isotherm is significantly less than for the 17° and 20° isotherms.

The results of the harmonic analysis of the 20°, 17° and 10° isotherms are given in table 10.

TABLE 10

Isotherm	Mean Depth	24^h Wave		12^h Wave		8^h Wave	
		A	α	A	α	A	α
20°	55.4	7.4	5.2	1.8	5.8	5.6	5.2
17°	430.4	27.5	16.6	14.0	5.2	9.4	0.9
10°	802.4	24.7	17.4	4.1	−1.6	2.3	6.4

The amplitudes are indicated by A and the phases by α.

Summary and Discussion of the Analysis

The 24 lunar hour wave was most pronounced and its amplitude attained significant proportions in the principal thermocline layer and its phase increased with increasing depth (table 10). The 12 lunar hour wave had an amplitude of twenty to fifty per cent that of the daily wave, and it attained its maximum at the same depth as the 24 lunar hour wave. The 8 lunar hour wave showed its greatest relative significance at the depth of the 20° isotherm where its amplitude was more than three times that of the 12 lunar hour wave; but reached its maximum amplitude at the depth of the 17° isotherm where it was relatively less important, being only 67 per cent of the 12 hour amplitude. The presence of an additional wave having a period of approximately 5 lunar hours, suggested by the final residue of the 10° isotherm (Fig. 6 and table 9), cannot be properly discussed at this time. It may be concluded tentatively that the 24 and 12 lunar hour waves are connected with the diurnal and semidiurnal tidal waves, but no tidal significance (as far as is known) can be attached to an 8 or 5 lunar hour wave. However, within reasonable limits, vertical oscillations of the 20°, 17°, and 10° isotherms at station 2639 can be expressed by equations 5, 9, and 13 for the 24 hour period beginning 15^h 34′, July 12, 1936.

The close agreement of extreme positions of the 20°, 17°, and 10° isotherms calculated from equations 5, 9, and 13 with actual observation during the 24 hour period beginning 15^h 34′, July 12, is illustrated by table 11.

TABLE 11

Isotherm	Calculated Range	Maximum Displacement	Observed Range	Maximum Displacement	Average Deviation All Values
20°	44.1– 70.0 (17^h) (5^h)	25.9	42– 76 (17^h)(5^h)	34	2.2
17°	407.5–481.1 (4^h) (16^h)	73.6	408–482 (5^h)(17^h)	74	1.9
10°	774.7–822.3 (4^h) (16^h)	47.6	777–830 (5^h)(17^h)	53	5.3

Comparison of extreme isotherm positions, calculated and observed. Figures in parentheses represent the approximate number of lunar hours after the lower culmination of the moon.

Since harmonic analysis of oscillations of the 20°, 17°, and 10° isotherms were carried out on data obtained during the final 24 hour period (page 19), a general idea of the applicability of the foregoing results to other twenty-four hour periods during the same time of year may be obtained by examining the depth changes of those isotherms observed during the previous $3\frac{1}{4}$ days at station 2639. This comparison is made in table 12 where the depths of the 20°, 17°, and 10° isotherms as observed between July 9 and July

TABLE 12

1936 Date	Lunar Time	July 9–12 $Z_{20°}$	$Z_{17°}$	$Z_{10°}$	July 12–13 $Z'_{20°}$	$Z'_{17°}$	$Z'_{10°}$	Theoretical $Z''_{20°}$	$Z''_{17°}$	$Z''_{10°}$	Absolute Differences $Z_{20°}-Z'_{20°}$	$Z_{17°}-Z'_{17°}$	$Z_{10°}-Z'_{10°}$	$Z_{20°}-Z''_{20°}$	$Z_{17°}-Z''_{17°}$	$Z_{10°}-Z''_{10°}$
July 9	17.7	74	450	839	44	475	823	45	477	822	30	25	16	29	27	17
" 9	23.7	62	430	838	51	418	806	54	414	811	11	12	22	8	16	27
" 9	6.1	76	450	839	70	410	782	69	411	778	6	40	59	7	39	61
" 10	12.2	82	440	829	54	421	812	55	422	810	28	19	17	27	18	19
" 10	0.4	56	422	834	53	414	799	52	415	801	3	8	35	4	7	33
" 10	6.6	57	425	841	66	412	785	66	414	780	9	13	56	9	11	61
" 11	13.6	58	413	833	53	435	809	55	438	817	5	22	24	5	25	16
" 11	19.9	66	415	824	52	443	802	54	442	822	14	28	22	12	27	2
" 11	1.9	72	422	830	57	410	787	55	414	784	15	12	43	17	8	46
" 12	7.9	52	433	823	56	419	786	58	421	785	4	14	37	6	12	38
" 12	14.4	60	424	—	53	453	—	52	452	—	7	29	—	8	28	—
										Average	12	20	33	12	20	32

Depths of 20°, 17° and 10° isotherms at station 2639. Lunar time refers to the number of lunar hours which have elapsed since the moon's last lower culmination. Z indicates observed depths of isotherms during July 9 to 12, Z' the scaled depths (for the same lunar time) for the 24 hour period beginning $15^h 30'$ on July 12, and Z'' the theoretical depth for the 24 hour period as given by equations 5, 9 and 13 for the 20°, 17° and 10° isotherms, respectively. Differences $Z_{20°}-Z'_{20°}$ etc. are absolute values.

12 are entered; time being given as the number of lunar hours since the moon's last lower culmination. The depths of these isotherms during the twenty-four hour period beginning $15^h\ 30'$, July 12, to correspond to the same lunar times are entered along with the theoretical values obtained from equations 5, 9, and 13. The absolute differences between values clearly illustrate that identical repetitions of vertical oscillations of the isotherms cannot be expected to take place from one lunar day to the next, and, hence, the theoretical equations (5, 9, and 13) which, within reasonable limits, express the vertical oscillations of the 20°, 17°, and 10° isotherms for the 24 hour period beginning $15^h\ 30'$ (L.C.T.), July 12, are unique for that time interval.

Thus, it is evident that our observations at station 2639 are not sufficient to give a representative picture of the short period oscillations, and, for the present, at least, their chief value must be confined to a rough measure of the order of magnitude of variations for oceanographic characteristics at fixed levels that may be expected in the sea during short periods. Similar situations have been shown to exist elsewhere, as for instance, in the Faero-Shetland Channel where the phases and amplitudes of diurnal and semi-diurnal waves, connected with the vertical oscillations vary in both horizontal and vertical directions; that is, they change with depth and from station to station within the same locality (Helland-Hansen, 1930).

With regard to further circumstances surrounding this immediate problem, and in order to illustrate possible differences in the causes of periodic variations of oceanographic properties at fixed levels in marine areas, where different hydrographic conditions prevail, the opinions of two authors will be discussed briefly.

Helland-Hansen (1930) after examining available data on short period oscillations from the North Atlantic suggested that the variations of temperature, and other oceanographic characteristics, at fixed levels in the sea may, to a great extent, be accounted for by horizontal transport of water by tidal currents, "since", as this author states, "they correspond to tidal periods and appear to be most marked in areas where tidal currents as well as convection currents may be assumed to be relatively strong". The means by which such oscillations may be brought about have been suggested by Helland-Hansen thus: "If the velocity of the tidal current is great in a direction transverse to the main direction of the convection current the transport of water with the tides will make the sloping isopycnal, isothermal, and isohaline surfaces move laterally for quite a distance. Repeated series of observations at a fixed station within the domains of the convection current will then reveal considerable variations in the depth of, for instance, the isothermal surfaces."

Helland-Hansen's analysis of short period oscillations in the Faero-Shetland Channel[7] strongly suggest that this explanation may, at least in part, represent the true course of such variations in regions where there is a sharp sloping of isopycnals and other surfaces. However, the available data from the North Atlantic at that time were not sufficient to extend this generalization to the open ocean; and results from station 2639 show that tilting of the temperature, salinity, and oxygen surfaces in the area is too small for horizontal displacements of the water layers to account for the observed vertical changes in positions of the isotherms in that region as emphasized on page 17.

A second investigation into the possibility that horizontal water movements, associ-

[7] Danish station: 61°27′N, 4°33′W, May 23–26, 1916, 780 meters depth, 67 hours duration (Knudsen, 1911); Scottish station: 61°32′N, 4°19′W, Aug. 13–14, 1916, 725 meters depth, more than 24 hours duration; "Michael Sars" station No. 115: 61°0′N, 2°41′W, Aug. 13–14, 1910, 580 meters depth, more than 24 hours duration.

ated with the passing of the tidal wave, may explain observed periodic variations of oceanographic factors at fixed levels has been made by Defant (1932) with reference to the upper 250 meters at "Meteor" station 288 (12° 37.6′N, 47° 36.1′W). This station differs essentially from "Atlantis" station 2639, first, because the principal thermocline (sprungschicht) begins at a depth of approximately 70 meters (instead of 400 meters), and second, because it lies in a region characterized by stronger horizontal gradients of temperature, salinity, oxygen, etc., so that "Meteor" station 288 is in a more favorable situation for its properties at fixed levels to fluctuate on account of horizontal movements of the water masses. In spite of this, however, Defant's conclusion, which is essentially the same as has been concluded for station 2639, is that the magnitude of the pure translatory motions of tidal currents which would be required to produce the observed variations of oceanographic characteristics at station 288 (about 300 cms/sec) are outside of the realm of probability.

Thus, on all grounds there is no escape from the conclusion that there are in the open ocean actual vertical movements of the water column which produce continuous variations in oceanographic characteristics at fixed levels. And, further to a large extent these vertical movements are associated with the tidal wave.

THE EFFECT OF VERTICAL OSCILLATIONS OF THE WATER COLUMN ON OCEANOGRAPHIC INVESTIGATIONS

Previous discussion has demonstrated the fact that at fixed levels throughout the entire water column at station 2639 variations of oceanographic characteristics ($t°$, $S\,°/_{oo}$, O_2, etc.) were continually taking place and the magnitudes of these variations are significant since they generally by far exceed the probable errors of the observations themselves. Hence, the question naturally arises as to the effect of vertical oscillations of the water column on the significance of conclusions derived from the usual oceanographic investigations.

Thus, while the present analysis is not based on sufficient data to warrant the use of the results to approximate the effect of vertical oscillations at station 2639 beyond the period of observation (page 5), it is possible from the standpoint of practical oceanography to ascertain the order of magnitude of variations which in general can be expected to take place at fixed levels from which can further be demonstrated the degree of disparity to be expected in various oceanographic calculations on account of the existence of vertical oscillations. The following discussion is intended to serve as a guide to the reliability of oceanographic calculations applied to data from the open sea, and to the rigidity with which results obtained from the region represented by station 2639 can be accepted.

The Total Variation of Temperature

Table 13 gives in tabular form the results of the entire series of temperature observations, scaled to standard depths, obtained at station 2639 during $4\frac{1}{4}$ days of continuous operation (July 9 to July 13, 1936). The values of:

$$t° \max - t° \min$$

and the arithmetic mean:

$$A = \frac{\sum_{i=1}^{n} f_i X_i}{N}$$

TABLE 13

Depth	A	B	C	D	E	F	G	H	I	J	K
0	25.0 13h57′	25.18 20h11′	25.10 02h53′	09h13′	21h55′	25.0 04h19′	11h35′	25.1 18h01′	25.1 00h22′	25.0 06h33′	25.0 13h15′
50	21.47 13h57′	20.75 20h11′	21.44 02h53′	22.20 09h13′	20.31 21h55′	20.36 04h19′	20.51 11h35′	21.08 18h01′	21.96 00h22′	20.30 06h33′	21.0 13h15′
100	18.82 13h58′	18.58 20h11′	19.03 02h53′	18.83 09h13′	18.23 21h55′	18.31 04h19′	18.34 11h35′	18.32 18h01′	18.56 00h22′	18.29 06h33′	18.19 13h15′
200	17.95 13h58′	17.94 20h12′	17.98 02h54′	17.99 09h14′	17.79 21h56′	17.80 04h20′	17.77 11h36′	17.72 18h02′	17.79 00h23′	17.80 06h34′	17.71 13h16′
300	17.70 13h58′	17.67 20h12′	17.76 02h54′	17.75 09h14′	17.52 21h56′	17.60 04h20′	17.51 11h36′	17.49 18h02′	17.58 00h23′	17.56 06h34′	17.58 13h16′
400	17.36 13h58′	17.20 20h12′	17.39 02h54′	17.29 09h14′	17.19 21h57′	17.19 04h21′	17.06 11h36′	17.10 18h03′	17.20 00h24′	17.24 06h34′	17.19 13h16′
500	16.50 13h59′	16.41 20h13′	16.47 02h55′	16.53 09h15′	16.28 21h57′	16.38 04h21′	16.17 11h37′	16.24 18h03′	16.19 00h24′	16.38 06h35′	16.13 13h17′
600	15.05 13h59′	15.12 20h13′	15.08 02h55′	15.33 09h15′	14.89 21h57′	14.97 04h21′	14.85 11h37′	14.62 18h04′	14.66 00h25′	15.01 06h35′	
700	13.12 14h00′	13.13 20h14′	13.21 02h56′	13.31 09h16′	12.82 21h58′	13.04 04h22′	12.85 11h38′	12.62 18h04′	12.79 00h25′	12.98 06h35′	
800	10.90 14h00′	10.87 20h14′	10.89 02h56′	10.65 09h16′	10.85 21h58′	10.84 04h22′	10.65 11h38′	10.48 18h05′	10.59 00h26′	10.51 06h36′	
900	8.79 14h01′	8.73 20h15′	8.68 02h57′	8.60 09h17′	8.50 21h59′	8.73 04h23′	8.54 11h38′	8.58 18h05′	8.64 00h26′	8.66 06h36′	
1000	7.02 14h01′	7.09 20h15′	7.08 02h57′	6.86 09h17′	6.85 21h59′	6.98 04h23′	6.90 11h39′	7.10 19h15′	7.16 01h35′	6.92 06h37′	
1100	5.82	5.87	5.90	5.94	6.00 23h08′	6.02 05h43′	6.17 13h01′	6.02 19h15′	6.10 01h35′	5.89 07h42′	5.94 14h32′
1200	5.17 15h08′	5.18 21h09′	5.27 04h10′	5.21 13h53′	5.35 23h08′	5.12 05h43′	5.34 13h02′	5.40 19h15′	5.39 01h36′	5.28 07h43′	5.62 14h32′
1300	4.80 15h09′	4.77 21h10′	5.00 04h10′	4.77 13h53′	4.88 23h09′	4.90 05h44′	4.91 13h02′	4.90 19h16′	4.99 01h36′	4.86 07h43′	4.92 14h33′
1400	4.45 15h09′	4.56 21h10′	4.63 04h11′	4.52 13h54′	4.59 23h09′	4.62 05h44′	4.62 13h02′	4.60 19h16′	4.81 01h36′	4.64 07h44′	4.66 14h33′
1500	4.33 15h09′	4.27 21h10′	4.38 04h11′	4.22 13h54′	4.37 23h10′	4.35 05h44′	4.37 13h03′	4.35 19h16′	4.44 01h37′	4.34 07h44′	4.30 14h33′
1600	4.10 15h10′	4.14 21h11′	4.12 04h12′	4.11 13h55′	4.16 23h10′	4.14 05h45′	4.14 13h03′	4.12 19h17′	4.24 01h37′	4.08 07h45′	4.22 14h34′
1800	3.77 15h10′	3.77 21h12′	3.79 04h13′	3.83 13h56′	3.82 23h11′	3.80 05h45′	3.93 13h04′	3.80 19h18′	3.88 01h38′	3.83 07h46′	3.85 14h34′
2000	3.64 15h11′	3.63 21h12′	3.63 04h13′	3.63 13h56′	3.61 23h11′	3.62 05h46′	3.64 13h05′	3.65 19h19′	3.65 01h38′	3.70 07h47′	3.68 14h35′
2250	3.52 15h12′	3.52 21h13′	3.47 04h14′	3.51 13h57′	3.50 23h12′	3.48 05h47′	3.46 13h05′	3.50 19h20′	3.50 01h39′	3.48 07h47′	3.49 14h36′
2500	3.36 15h13′	3.36 21h14′	3.33 04h15′	3.37 13h58′	3.37 23h13′	3.33 05h48′	3.35 13h06′	3.35 19h20′	3.35 01h40′	3.38 07h48′	3.34 14h37′
2750	3.18 15h14′	3.19 21h15′	3.17 04h16′	3.18 13h59′	3.16 23h14′	3.18 05h49′	3.16 13h07′	3.17 19h21′	3.16 01h40′	3.20 07h48′	3.18 14h38′
3000	3.01 17h47′	3.03 00h05′	2.98 06h39′	2.96 18h05′	3.02 01h36′	2.98 08h17′	2.99 15h19′	2.97 21h37′	2.97 03h58′		2.96 16h54′
3250	2.78 17h48′	2.79 00h06′	2.77 06h40′	2.77 18h05′	2.81 01h37′	2.84 08h18′	2.82 15h20′	2.79 21h38′	2.77 03h59′		2.81 16h55′
3500	2.59 17h49′	2.56 00h07′	2.57 06h41′	2.57 18h06′	2.61 01h38′	2.63 08h19′	2.61 15h21′	2.60 21h39′	2.58 04h00′		2.62 16h56′
3750	2.47 17h50′	2.45 00h08′	2.46 06h42′	2.46 18h07′	2.47 01h39′	2.47 08h20′	2.48 15h22′	2.45 21h40′	2.45 04h01′		2.47 16h57′
4000	2.42 17h50′	2.41 00h09′	2.41 06h43′	2.39 18h08′	2.40 01h40′	2.42 08h21′	2.44 15h23′	2.41 21h40′	2.41 04h01′		2.41 16h57′
4250	2.36 17h51′	2.36 00h09′	2.39 06h44′	2.36 18h09′	2.38 01h40′	2.38 08h22′	2.39 15h23′	2.38 21h41′	2.39 04h02′		2.36 16h58′
4500	2.34 17h52′	2.35 00h10′	2.36 06h45′	2.34 18h10′	2.38 01h41′	2.35 08h22′	2.36 15h24′	2.36 21h42′	2.36 04h03′		2.34 16h59′
4750	2.33 17h53′	2.33 00h11′	2.34 06h45′	2.33 18h10′	2.35 01h42′	2.35 08h23′	2.36 15h25′	2.35 21h43′	2.35 04h04′		2.33 17h00′
5000			2.35 06h46′		2.35 01h43′	2.33 08h24′	2.35 15h26′	2.35 21h44′	2.33 04h05′		2.33 17h01′
Date	7/9	7/9–10	7/10	7/10	7/10–11	7/11	7/11	7/11	7/12	7/12	7/12

Depth	L	M	N	O	P	Q	R	S	T	U	V	W	$t°_{max} - t°_{min}$	Mean $t°$
0	20h24′	22h30′	00h32′	02h26′	04h33′	25.1 06h29′	08h30′	10h35′	12h29′	14h22′	25.3 16h36′	20h01′	0.30	25.09
50	20.78 20h24′	19.93 22h30′	20.50 00h32′	20.51 02h26′	21.94 04h33′	20.70 06h29′	20.21 08h30′	20.36 10h35′	20.32 12h29′	20.35 14h22′	19.54 16h36′	20.10 20h01′	2.66	20.72
100	18.48 20h25′	18.25 22h30′	18.41 00h32′	18.48 02h26′	18.91 04h33′	18.76 06h29′	18.50 08h30′	18.51 10h35′	18.50 12h30′	18.42 14h23′	18.46 16h37′	18.45 20h02′	0.84	18.51
200	17.74 20h25′	17.71 22h31′	17.69 00h33′	17.69 02h27′	17.68 04h34′	17.73 06h30′	17.73 08h31′	17.68 10h36′	17.78 12h31′	17.81 14h23′	17.89 16h37′	17.84 20h03′	0.31	17.79
300	17.57 20h26′	17.44 22h31′	17.40 00h33′	17.42 02h27′	17.41 04h34′	17.46 06h30′	17.45 08h31′	17.49 10h36′	17.52 12h31′	17.62 14h24′	17.64 16h38′	17.60 20h03′	0.36	17.55
400	17.19 20h27′	17.12 22h32′	17.08 00h34′	17.09 02h28′	17.06 04h35′	17.13 06h30′	17.15 08h31′	17.11 10h37′	17.17 12h32′	17.31 14h25′	17.37 16h39′	17.24 20h04′	0.33	17.19
500	16.11 20h27′	16.15 22h32′	16.03 00h34′	16.09 02h28′	16.06 04h35′	15.95 06h31′	16.09 08h32′	16.17 10h37′	16.28 12h33′	16.65 14h25′	16.85 16h39′	16.56 20h04′	0.90	16.29
600	14.55 20h28′	14.69 22h33′	14.71 00h35′	14.67 02h28′	14.85 04h35′	14.77 06h31′	14.88 08h32′	14.66 10h38′	14.81 12h33′	15.22 14h26′	15.57 16h40′	14.98 20h05′	1.02	14.91
700	12.74 20h28′	12.90 22h33′	12.96 00h35′	12.88 02h29′	12.98 04h36′	12.90 06h31′	12.84 08h32′	12.81 10h38′	13.11 12h34′	13.21 14h27′	13.76 16h41′	13.08 20h06′	1.14	13.04
800	10.69 20h29′	10.31 22h34′	10.72 00h36′	10.69 02h29′	10.53 04h36′	10.68 06h32′	10.68 08h33′	11.01 10h39′	11.29 12h34′	11.06 14h27′	11.73 16h41′	11.03 20h06′	1.42	10.80
900	8.22 20h29′	8.19 22h34′	8.57 00h36′	8.48 02h30′	8.66 04h36′	8.73 06h32′	8.73 08h33′	8.78 10h39′	9.14 12h35′	9.20 14h28′	9.35 16h42′	8.71 20h07′	1.16	8.69
1000	7.09 20h30′	6.84 22h34′	7.04 00h36′	7.00 02h30′	7.09 04h37′	7.17 06h32′	6.99 08h33′	6.98 10h39′	7.14 12h36′	7.31 14h29′	7.69 16h43′	7.03 20h07′	0.85	7.06
1100	6.20 20h30′	5.97 22h35′	6.00 00h37′	5.98 02h30′	6.07 04h37′	6.06 06h33′	5.91 08h34′	5.94 10h40′		6.17 14h29′	6.61 16h43′	6.06 20h08′	0.79	6.03
1200	5.29 20h31′	5.24 22h35′	5.39 00h37′	5.44 02h31′	5.61 04h37′	5.62 06h33′	5.33 08h34′	5.37 10h40′		5.36 14h30′		5.54 20h08′	0.50	5.36
1300		4.71 22h36′	4.88 00h38′	4.82 02h31′	4.89 04h38′	4.81 06h34′				4.84 14h30′			0.29	4.86
1400			4.50 00h38′	4.48 02h31′	4.51 04h38′					4.54 14h31′			0.36	4.58
1500													0.22	4.34
1600													0.16	4.14
1800													0.16	3.82
2000													0.09	3.64
2250													0.06	3.49
2500													0.05	3.35
2750													0.04	3.18
3000													0.07	2.99
3250													0.07	2.80
3500													0.07	2.59
3750													0.03	2.46
4000													0.05	2.41
4250													0.03	2.38
4500													0.04	2.35
4750													0.03	2.34
5000													0.02	2.34
Date	7/12	7/12	7/13	7/13	7/13	7/13	7/13	7/13	7/13	7/13	7/13	7/13		

Temperature data from station 2639 reduced to standard depths. Time is referred to Greenwich meridian, to convert to local civil time of mean station position subtract 4 hours and 26 minutes.

for each individual level are tabulated in the last two columns of table 13. The mean values are plotted against depth in figure 3.

Comparison of table 13 with figure 3 illustrates that maximum temperature variations invariably occur in the thermocline. At the surface, during the $4\frac{1}{4}$ day interval, the value of $t^\circ_{max} - t^\circ_{min}$ was only 0.30°, but at 50 meters depth, which lies in the summer thermocline, this value had increased to 2.66°; while below, in the relatively homogeneous water between 200 and 400 meters variations of only 0.31° to 0.36° were obtained. On entering the principal thermocline, which approximately lies between 400 and 1200 meters, the $t^\circ_{max} - t^\circ_{min}$ variations again become great, increasing from 0.90° at 500 meters depth to a maximum value of 1.42° at 800 meters depth and then falling off continuously to 0.79° at 1100 meters and to 0.50° at 1200 meters depth. In still deeper water below the thermocline, $t^\circ_{max} - t^\circ_{min}$ values continued to decrease with decreasing vertical temperature gradient; between 1400 and 2000 meters the values declined from 0.36° to 0.09° and continued to diminish still deeper until at 5000 meters depth the $t^\circ_{max} - t^\circ_{min}$ value was 0.02°, or the approximate error of the thermometers (page 6).

Correlation of Salinity and Temperature

The 250 salinity determinations between depths of 100 and 5000 meters, at station 2639, are plotted as a function of temperature in figure 7. The majority of observations can be fitted by a smooth curve having a width not exceeding 0.02 °/₀₀ (page 6). Depths corresponding to their respective mean temperatures are indicated along the curve.

It is clear from the curve, $S = f(t)$, that each water particle retains, within reasonable limits, its temperature and salinity values as it oscillates up and down. Vertical displacements of the water column will not alter the temperature-salinity correlation and changes in temperature or salinity at fixed levels will be in accordance with the relation $S = f(t)$.

Dynamic Calculations

To illustrate by means of actual numerical values the order of magnitude of the discrepancy which may arise in calculations of the dynamics of the sea, because of temperature and salinity variations at fixed levels, use has been made of the fact that at station 2639 (table 13) the greatest temperature differences at fixed levels between surface and 5000 meters depth occurred between series C (22^h 27′, July 9 to 2^h 20′, July 10, L.C.T.) and H (13^h 35′ to 17^h 18′, July 11, 1936, L.C.T.); and these two series will now be treated as two separate oceanographic stations.

Anomaly of Dynamic Height and Current Calculations

This calculation which is intended to illustrate the variance in results of dynamic height anomaly for station 2639 was carried out by the method of V. Bjerknes and is facilitated by the use of tables compiled by Bjerknes and his coworkers (1912).

The anomaly of dynamic height, Δd, is symbolically expressed:

$$\Delta d = d - d_n = \int_0^p (\alpha - \alpha_n) dp = \int_0^p \alpha_a \, dp$$

where, d, is the dynamic height from a specified reference level (2000 meters); p, is the pressure; d_n, the dynamic height from the same surface but in a sea where the salinity is everywhere 35.00 °/₀₀ and the temperature 0°C; α, is the specific volume, and, α_n, the specific volume at 35.00 °/₀₀ and 0°C.

Results of the calculations of dynamic height anomalies for each layer, and the summation of Δd between 2000 meters and the surface, for each of series C and H, are given in table 14. The final column sums the differences in anomaly of dynamic height and shows a total difference of 8.45 dynamic centimeters between 2000 meters and the surface for the two series. Since either series C or H may represent station 2639, the indication is that, because of vertical oscillations of the water column, significant errors may occur in dynamic calculations applied to the sea.

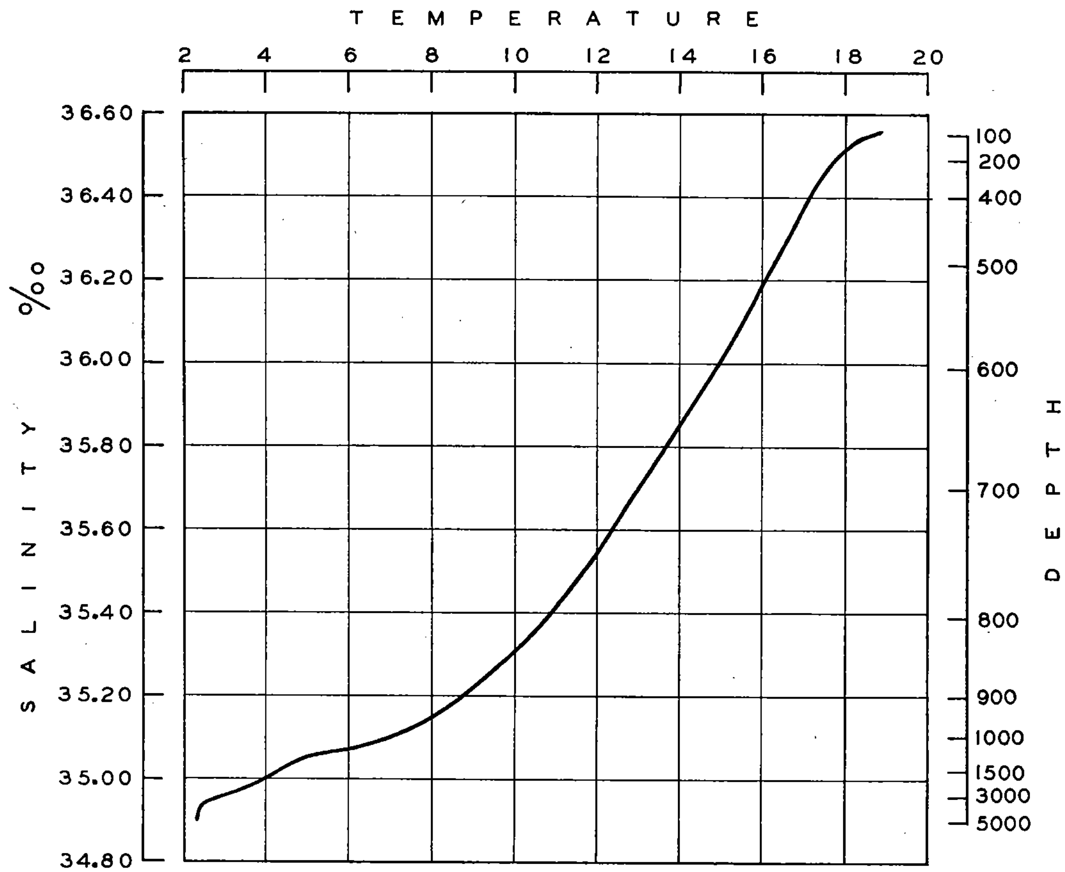

FIG. 7.—Temperature salinity correlation at "Atlantis" station 2639.

As a numerical illustration of the discrepancy which will be produced in oceanic current calculations by a change of 8.45 dynamic centimeters, the following hypothetical example is given.

The velocity normal to a line between two verticals in the sea, relative to a reference level where the pressure is everywhere uniform, is given by:[8]

$$V \text{ cms sec}^{-1} = \frac{D \times 10^5}{2\omega L \sin \phi}$$

where D is the difference in dynamic height (dynamic meters) between the two verticals

[8] For an excellent discussion of the circulation theorem the reader is referred to McEwen (1932).

as measured from a reference level (e.g., 2000 meters), ω is the angular velocity of the earth's rotation, ϕ is the mean latitude, and L is the horizontal distance between the verticals. Since oceanographic stations for current calculation are frequently spaced 40 to 55 kilometers apart, by substituting appropriate values in the above equation ($2\omega \sin 35^\circ = 8.36 \times 10^{-5}$ sec^{-1}, $D = 0.0845$ dynamic meters, $L = 40 \times 10^5$ cms) a velocity:

$$V = \frac{8.45 \times 10^3}{8.36 \times 40} = 25.3 \text{ cms sec}^{-1}$$

is obtained. This then represents the amount of the discrepancy which will be introduced into dynamic current calculations, under conditions outlined above, by a change of 8.45

TABLE 14

Depth (meters)	Anomaly Dynamic Height Series C	Series H	Anomaly C–H	Σ(C–H) (centimeters)
0	15.25	14.85	0.40	8.45
50	10.90	10.25	0.65	8.05
100	17.60	16.20	1.40	7.40
200	16.60	15.90	0.70	6.00
300	16.50	16.00	0.50	5.30
400	16.20	15.80	0.40	4.80
500	15.30	14.90	0.40	4.40
600	14.00	13.40	0.60	4.00
700	12.40	11.50	0.90	3.40
800	10.30	9.40	0.90	2.50
900	8.30	8.00	0.30	1.60
1000	6.80	6.80	0.00	1.30
1100	5.90	5.90	0.00	1.30
1200	5.50	5.20	0.30	1.30
1300	5.20	4.90	0.30	1.00
1400	4.80	4.80	0.00	0.70
1500	4.70	4.60	0.10	0.70
1600	9.20	8.80	0.40	0.60
1800	9.20	9.00	0.20	0.20
2000				0.00

dynamic centimeters in the difference between the dynamic heights of two verticals in the sea. Certainly, nothing more need be said of the significant effect of vertical oscillations in the sea on the calculations of both dynamic typography of the sea surface and the computed current velocities.

The Transport of Water Between two Verticals in the Sea

To carry the discussion a step further in order to elucidate the effect vertical oscillations may have on calculations of water movements in the sea, the results of two separate determinations of the water transport between the verticals defined by "Atlantis" stations 2639 (series C and H) and 1226 (35° 07′N, 71° 53′W; April 21, 1932) have been made; station 1226 being located on the eastern edge of the Gulf Stream off Chesapeake Bay. The two sets of observations for station 2639 (represented by series C and H) may be considered as random since either set of data might have been obtained by the usual oceanographic procedure and specified as representative for this station. Since stations 1226 and 2639 are separated by a time interval of more than four years the results have only relative values.

The method used for this calculation was developed by Werenskiold (1935) to compute transports of water in coastal currents from temperature and salinity determinations at a single station. Thus, if the function V (equation 14) is computed for two verticals in the open ocean, the total transport of water between them is given by the difference between the two values of V. Hence, by calculating V for station 1226 and for series C and series H of station 2639, the variance of the results:

$$V_{2639\mathrm{C}} - V_{1226}$$

and

$$V_{2639\mathrm{H}} - V_{1226}$$

illustrates the significance of short period vertical oscillations of the water column on calculations of water transport in the open ocean.

In the equation:

$$V = \frac{g}{2\lambda\alpha_0}\int_0^{H^2}(\alpha - \alpha_0)dZ^2 \tag{14}$$

V=volume transport, cm³ sec⁻¹; g=acceleration of gravity, cms sec⁻², $\lambda=2\omega \sin \phi$; ω=angular velocity of earth, sec⁻¹; ϕ=latitude; α_0=specific volume of the reference level, which in the case under consideration was taken at 2000 meters.[9]

To adapt equation 14 to this investigation we note that specific volume at any level, H, above the reference level is $\alpha=\alpha_n+\alpha_a$, where α_n is specific volume at the depth H of sea water of 35.00 °/₀₀ salinity and 0°C, and, α_a, the anomaly of specific volume calculated as on page 29. Similarly, at the reference level the designation is: $\alpha_0=\alpha_{n_0}+\alpha_{a_0}$. Thus, we may write:

$$(\alpha - \alpha_0) = (\alpha_n + \alpha_a) - (\alpha_{n_0} + \alpha_{a_0}) = \alpha_a - \alpha_{a_0} + \alpha_n - \alpha_{n_0} \tag{15}$$

and equation 14 becomes:

$$V = \frac{g}{2\lambda\alpha_0}\left[\int_0^{H^2}(\alpha_a - \alpha_{a_0})dZ^2 + \int_0^{H^2}(\alpha_n - \alpha_{n_0})dZ^2\right]. \tag{16}$$

The first part of the integration is rapidly carried out for series C and H of station 2639 by making use of the previously calculated specific volume anomalies which form the basis for the calculation of the dynamic height anomalies in table 14 (page 31); for station 1226 a separate determination of the specific volume anomalies is required. The second part of the integration is the same for all three verticals.

The results of the transport calculations between surface and a 2000 meter reference level are summarized in table 15. The two transport values ($V_{2639\text{-C}}-V_{1226}$ and $V_{2639\text{-H}}-V_{1226}$) obtained between verticals represented by stations 1226 and 2639 illustrate the order of magnitude of disparity which may arise in a calculation of the total transport

TABLE 15

Station	$\int\alpha_a-\alpha_0$	$\int\alpha_n-\alpha_{n_0}$	$\int\alpha-\alpha_0$	$\frac{g}{2\lambda\alpha_0}$	V cm³ sec⁻¹	Transport cm³ sec⁻¹ $V_{2639\text{-C}}-V_{1226}$	Transport cm³ sec⁻¹ $V_{2639\text{-H}}-V_{1226}$
1226	879.1×10⁴	11,638×10⁴	12,517×10⁴	583×10⁴	7.297×10¹⁴	64×10¹¹	19×10¹¹
2639-C	987.9×10⁴	11,638×10⁴	12,626×10⁴	583×10⁴	7.361×10¹⁴		
2639-H	911.0×10⁴	11,638×10⁴	12,549×10⁴	583×10⁴	7.316×10¹⁴		

of water between two verticals in the sea. The results are somewhat startling in view of

[9] Werenskiold suggests a graphical method for the evaulation of this integral: the density or specific volume value is plotted against the square of the depth and the area under the curve measured, and, after reduction to the proper units, multiplied by the constant term. In this case, equation 14 reduces to: $V=g/2\lambda\alpha_0 A$, where A is the measured area.

TABLE 16

DEPTH	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	MEAN cc/L	$O_{2max}-O_{2min}$
0	4.74	4.69	4.66	4.69	4.79	4.69	4.69	4.69	4.68	4.68	4.71	4.61	4.68	4.68			4.68			4.61			4.64	4.68	0.18
50	5.06	5.25	5.22	5.11	5.42	5.45	5.45	5.39	5.22	5.44	5.18	5.26	5.44	5.38	5.36	5.24	5.31	5.34	5.40	5.31	5.46	5.29	5.14	5.31	0.40
100	5.00	5.04	5.18	5.31	5.07	5.00	5.12	5.23	5.26	4.90	4.89	5.07	4.83	5.00	4.91	5.11	5.09	4.99	5.02	5.20	4.71	5.06	4.94	5.04	0.60
200	4.81	4.86	4.84	4.72	4.74	4.76	4.80	4.78	4.74	4.77	4.74	4.68	4.68	4.74	4.75	4.70	4.74	4.73	4.60	4.74	4.56	4.77	4.69	4.74	0.30
300	4.57	4.62	4.62	4.66	4.63	4.77	4.64	4.69	4.71	4.68	4.60	4.65	4.61	4.72	4.65	4.68	4.71	4.68	4.62	4.64	4.68	4.69	4.58	4.65	0.20
400	4.55	4.48	4.51	4.51	4.62	4.59	4.48	4.66	4.58	4.62	4.51	4.51	4.56	4.47	4.48	4.49	4.60	4.52	4.42	4.54	4.61	4.68	4.54	4.54	0.26
500	4.18	4.21	4.16	4.22	4.32	4.28	4.21	4.27	4.22	4.31	4.12	4.21	4.17	4.23	4.26	4.24	4.22	4.18	4.22	4.19	4.39	4.40	4.32	4.24	0.28
600	4.12	4.06	4.07	4.07	3.95	4.04	4.07	4.02	4.00	4.00	3.97	4.08	4.07	3.99	3.99	3.97	4.03	4.08	3.99	3.92	4.11	4.12	4.06	4.03	0.20
700	3.64	3.77	3.69	3.76	3.65	3.78	3.86	3.77	3.67	3.81		3.73	3.81	3.75	3.70	3.67	3.76	3.71	3.70	3.66	3.77	3.92	3.73	3.74	0.28
800	3.44	3.42	3.31	3.34	3.47	3.38	3.48	3.50	3.41	3.41		3.38	3.48	3.52	3.48	3.42	3.31	3.33	3.44	3.45	3.43	3.52	3.39	3.42	0.21
900	3.54	3.58	3.59	3.62	3.68	3.53	3.58	3.56	3.51	3.59		3.57	3.66	3.56	3.52	3.33	3.47	3.51	3.35	3.39	3.35	3.44	3.43	3.51	0.35
1000	3.87	4.06	4.10	4.12	4.11	4.06	4.06	4.06	4.00	4.16		4.07	4.13	4.03	4.01	3.96	3.90	4.09	4.02	3.94	3.94		3.99	4.03	0.29
1100	4.61	4.68	4.65	4.70	4.52	4.60	4.57	4.54	4.53	4.61	4.68	4.57	4.62	4.61	4.53	4.55	4.56	4.61	4.58		4.45		4.51	4.58	0.25
1200	5.18	5.12	5.10	5.17	4.98	5.13	5.03	4.96	4.98	5.06	4.91	5.04	5.08	4.94	4.96	4.88	4.91	4.95	4.92		4.91		4.86	5.00	0.32
1300	5.38	5.41	5.33	5.40	5.41	5.37	5.31	5.34	5.29	5.34	5.32		5.46	5.32	5.34	5.29	5.37				5.32			5.35	0.17
1400	5.57	5.48	5.53	5.57	5.49	5.53	5.51	5.55	5.42	5.47	5.48			5.57	5.59	5.55					5.57			5.53	0.17
1500	5.65	5.64	5.65	5.75	5.66	5.73	5.70	5.66	5.60	5.66	5.68													5.67	0.15
1600	5.79	5.83	5.82	5.77	5.82	5.89	5.82	5.83	5.72	5.82	5.85													5.81	0.17
1800	6.12	6.00	6.05	5.92	5.95	6.00	5.99	6.05	5.95	6.01	5.94													6.00	0.20
2000	6.21	6.06	6.12	6.10	6.11	6.09	6.12	6.07	6.00	6.01	5.96													6.08	0.25
2250	6.15	6.10	6.13	6.06	6.04	6.09	6.11	6.02	6.00	6.01	6.00													6.06	0.15
2500	6.00	6.05	6.06	6.02	6.02	6.04	6.00	6.00	6.00	5.94	5.95													6.01	0.12
2750	6.00	6.02	6.02	5.99	5.96	5.97	5.96	6.00	6.00	5.94	5.92													5.98	0.10
3000	6.00	6.02	5.99	5.96	5.98	5.99	5.96	5.97	5.97		5.92													5.98	0.10
3250	5.99	5.96	5.98	5.97	6.00	5.99	5.96	5.94	5.93		5.92													5.96	0.08
3500	5.95	5.96	5.98	5.98	5.99	6.04	5.96	5.93	5.93		5.93													5.96	0.11
3750	5.96	5.96	5.95	5.96	5.97	5.98	5.98	5.91	5.93		5.93													5.95	0.07
4000	5.94	5.96	5.92	5.93	5.95	5.97	5.98	5.94	5.90		5.92													5.94	0.08
4250	5.91	5.89	5.89	5.85	5.91	5.94	5.94	5.87	5.86		5.86													5.89	0.09
4500	5.89	5.82	5.86	5.82	5.85	5.86	5.90	5.84	5.75		5.80													5.84	0.15
4750	5.87	5.77	5.83	5.77	5.84	5.85	5.86	5.77	5.74		5.78													5.81	0.13
5000		5.71		5.77	5.93	5.78	5.80	5.89	5.72		5.75													5.79	0.22
Date	7/9	7/9–10	7/10	7/10	7/10–11	7/11	7/11	7/11	7/12	7/12	7/12	7/12	7/12	7/13	7/13	7/13	7/13	7/13	7/13	7/13	7/13	7/13	7/13		

Oxygen data from station 2639 reduced to standard depths; for sampling time see table 13 (page 28).

the fact that under the usual methods of oceanographic sampling either series C or H might have been considered representative of station 2639. The percental difference amounts to:

$$\frac{64 \times 10^{11} - 19 \times 10^{11}}{64 \times 10^{11}} = 70.3\%$$

THE EFFECT OF VERTICAL OSCILLATIONS OF THE WATER COLUMN ON OCEANOGRAPHIC INVESTIGATIONS INVOLVING OXYGEN OBSERVATIONS

Due to the present wide usage which is being made of oxygen observations in the ocean for both physical and biological oceanographical investigations the results of the preceding discussion are utilized to illustrate the significance of a continuously varying oxygen content at fixed levels in the sea.

The Total Variation of Oxygen

Table 16 gives the results of the entire series of oxygen determinations (scaled to standard depths) obtained at station 2639 during more than four days of continuous operation (July 9 to July 13, 1936). The values of the extreme variation of oxygen ($O_{2max} - O_{2min}$), the arithmetic mean (page 27) for individual levels are tabulated in the last two right hand columns of the table. Values of the arithmetic mean are plotted against depth in figure 4.

Values of $O_{2max} - O_{2min}$ greater than 0.15 cc per liter occurred in the upper 2250 meters; in deeper waters the range somewhat decreased, but for any particular level it was never less than 0.07 cc per liter. The maximum values of $O_{2max} - O_{2min}$, 0.30 to 0.60 cc per liter (confined to depths between 50 and 200 meters), coincided with the maximum vertical oxygen gradient in the layer of photosynthesis. As deep as 1200 meters the maximum oxygen variations for fixed levels were more than 0.20 cc per liter, and the diminution of $O_{2max} - O_{2min}$ values coincided with the decline of the vertical oxygen gradient with depth (Fig. 4).

The decided increase in values of $O_{2max} - O_{2min}$ between 4000 and 5000 meters (0.09–0.22 cc per liter) is puzzling and cannot be attributed to increases in amplitude of the vertical displacements at these depths since there is no increase in temperature variations. Between 4000 and 5000 meters the vertical oxygen gradient decreases somewhat ($dO_2/dZ = -0.5 \times 10^{-4}$ at 4000 meters; -2.1×10^{-4} at 4250 meters; and -1.4×10^{-4} at 4500 meters; but the decrease is not sufficient to account for the increased variations of oxygen at these depths.

Values of $O_{2max} - O_{2min}$ indicate in a general way the maximum amount of variation in oxygen content measured at a series of fixed levels, but the values by themselves may be misleading since they do not give any idea of the probability by which any observation will differ in value from any other observation when taken at random from the same fixed level. And, since a knowledge of the probability is important because of the various interpretations given to results of oxygen distribution in the sea, the oxygen variations at fixed levels are considered in more detail, and an estimate is made of the probability with which various degrees of differences occurred in the values of any two samples selected at random from the same fixed level. To carry out this analysis it was first necessary to obtain from the oxygen results for each level (table 16) the greatest possible number of differences between the individual observations. The frequencies of the differences obtained for each level were next grouped into classes, having an interval of 0.05 cc per liter, and the probability of the occurrence of these classes calculated. Thus, if the event can happen in m ways and fail in n ways, and, if each of these ways is equally likely to happen, the probability of its happening is:

$$p = \frac{m}{m + n}.$$

The results are given in table 17. In the last two columns of the table certain of the results are summarized and the probabilities that individual oxygen samples at the same fixed level may differ from each other by more than 0.10 cc per liter and by more than 0.20 cc per liter are calculated.

Table 17 has been prepared primarily to serve as a guide to the degree of discrepancy in oxygen measurements made in the water of the region surrounding station 2639 (66° 25′W, 35° 07′N), provided that the probable error of the oxygen determination is about 0.03 cc per liter. This table will serve in a general way as a means of estimating the validity of the oxygen results for various levels and will fix general limits on the rigidity of conclusions drawn from oxygen sampling; there is no reason to doubt that within reasonable limits it possesses a general validity which may be applied to oxygen investigations made in the surrounding region.

TABLE 17

Class Value / Depth	0.0–0.04	0.05–0.09	0.10–0.14	0.15–0.19	0.20–0.24	0.25–0.29	0.30–0.34	0.35–0.39	0.40–0.64	>0.10	>0.20
0	0.627	0.235	0.128	0.010						0.138	0.00
50	0.194	0.217	0.174	0.142	0.130	0.059	0.055	0.024	0.004	0.588	0.272
100	0.146	0.198	0.170	0.150	0.118	0.079	0.047	0.040	0.052	0.656	0.336
200	0.411	0.293	0.166	0.083	0.032	0.011	0.004			0.296	0.047
300	0.478	0.364	0.126	0.028	0.004					0.158	0.004
400	0.360	0.305	0.221	0.083	0.027	0.004				0.335	0.031
500	0.364	0.293	0.186	0.103	0.046	0.008				0.343	0.054
600	0.387	0.380	0.170	0.055	0.008					0.233	0.008
700	0.342	0.312	0.208	0.087	0.030	0.021				0.346	0.051
800	0.359	0.325	0.221	0.078	0.017					0.316	0.017
900	0.264	0.247	0.155	0.152	0.113	0.043	0.022	0.004		0.489	0.182
1000	0.281	0.300	0.210	0.138	0.052	0.019				0.419	0.071
1100	0.376	0.348	0.162	0.095	0.014	0.005				0.276	0.019
1200	0.229	0.252	0.186	0.148	0.114	0.057	0.014			0.519	0.185
1300	0.478	0.390	0.110	0.022						0.132	0.0
1400	0.476	0.362	0.114	0.048						0.162	0.0
1500	0.509	0.382	0.091	0.018						0.109	0.0
1600	0.528	0.291	0.163	0.018						0.181	0.0
1800	0.309	0.400	0.218	0.055	0.018					0.291	0.018
2000	0.327	0.273	0.273	0.073	0.036	0.018				0.400	0.054
2250	0.400	0.346	0.218	0.036						0.254	0.0
2500	0.527	0.382	0.091							0.091	0.0
2750	0.655	0.309	0.306							0.036	0.0
3000	0.756	0.222	0.022							0.022	0.0
3500	0.600	0.333	0.067							0.067	0.0
4000	0.800	0.200								0.00	0.0
4500	0.534	0.333	0.111	0.022						0.133	0.0
5000	0.334	0.296	0.148	0.148	0.074					0.370	0.074

Reference to table 17 will show that the greatest disparity among oxygen observations from a fixed level occurred at 50 and 100 meters where the highest probabilities (0.588–0.656) that two oxygen samples taken at random from the same fixed level may

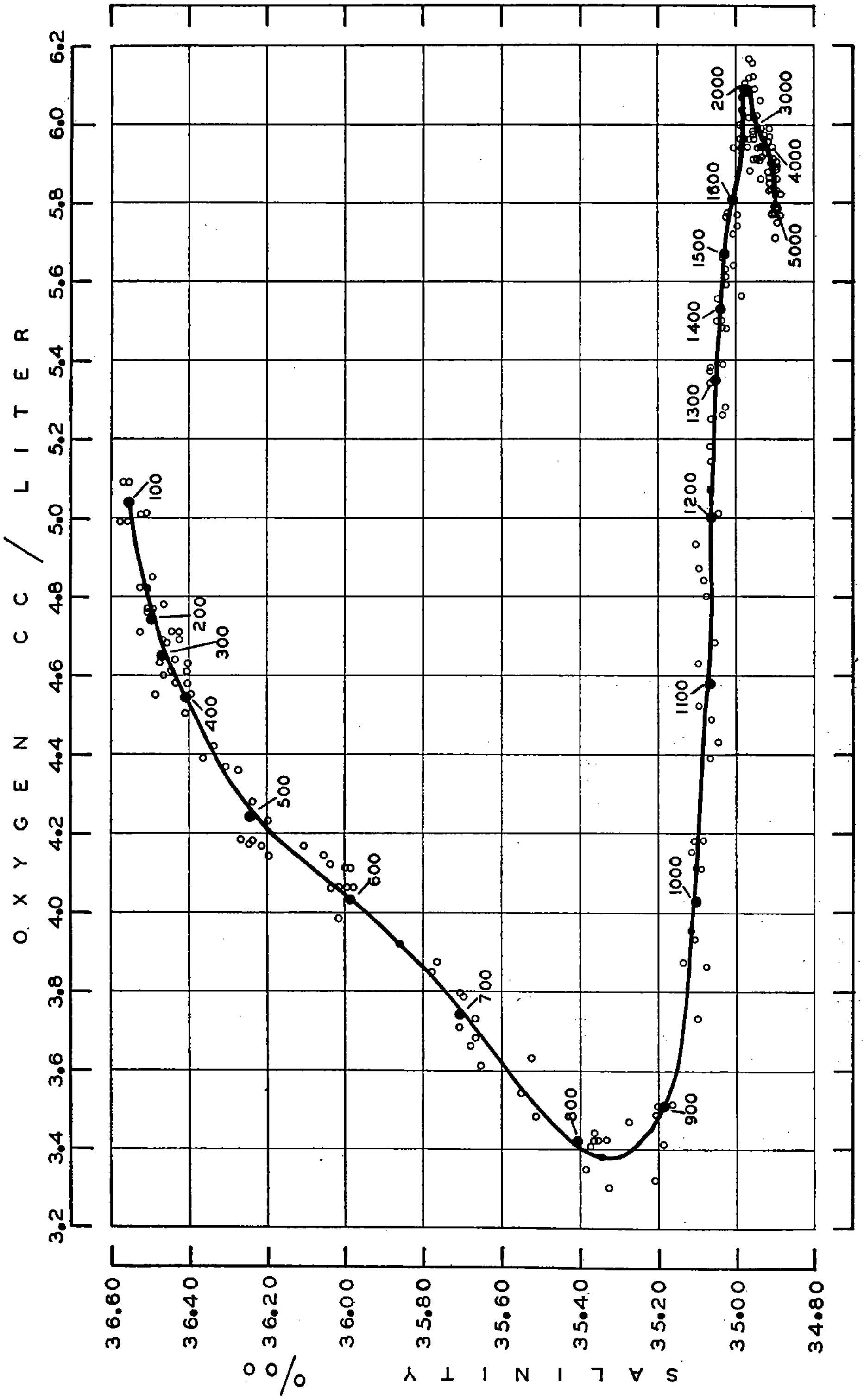

FIG. 8.—Oxygen salinity correlation at "Atlantis" station 2639.

differ from each other by more than 0.10 cc per liter. At depths of 50 and 100 meters there are two primary factors acting to produce wide differences among individual oxygen determinations, namely, the presence of the steep summer thermocline and the production of oxygen by photosynthesis. In deeper water, between 100 and 1000 meters, the probability of samples differing by more than 0.10 cc decreases to 0.158–0.489; between 1100 and 2000 meters it is between 0.109 and 0.519, and between 2250 and 5000 meters it is 0.00–0.370. Between depths of 4000 and 5000 meters the increase in the probability of the occurrence of differences of more than 0.10 cc per liter is associated with increased values of $O_{2max}-O_{2min}$; previously discussed on page 34.

Thus, the probability that individual oxygen determinations from the same fixed level owing to vertical displacements may differ from each other by 0.20 cc per liter, or more, is confined almost entirely to the water column above 1200 meters where the probability varies between 0.004 and 0.336, and, as in the previous case, the higher probabilities are restricted to the 50 and 100 meter levels. These results are, however, only tentative.

Oxygen Salinity Correlation

The results of 250 oxygen and salinity determinations at station 2639 are plotted as $O_2=f$ (S °/₀₀) in figure 8, and a depth scale fixed by mean values of the attributes is indicated along the curve. Oxygen salinity correlation in the sea, unlike that of temperature and salinity, is not conservative and for the same water mass will show a distinct horizontal variation since the concentration of oxygen is a function of the biochemical activity of the water. From a biochemical standpoint the oxygen content at depths below 100 to 200 meters is primarily altered only by respiratory processes which diminish it.

From a depth of about 100 meters both oxygen and salinity (Fig. 9) decrease to the depth of the oxygen minimum concentration (about 850 meters), and in still deeper water oxygen then increases with depth along with slightly decreasing salinity, to about 2000 meters where it reaches a secondary maximum. Short period vertical displacements of the water column will not by themselves disturb the oxygen salinity correlation, since individual water particles will retain their characteristics during vertical displacement (within reasonable limits), and the effect of vertical displacements will be manifested by time variations in the oxygen salinity ratios at fixed levels. The magnitude of such variations at fixed levels will depend on the relative rates of change of salinity and oxygen with depth, as well as on the amplitude of the vertical displacement.

"SEASONAL VARIATION" OF OXYGEN CONTENT AT MID DEPTHS WITH RESPECT TO SHORT PERIOD OXYGEN VARIATIONS AT FIXED LEVELS

One of the questions which oceanographic investigations have recently brought to light is that concerning the seasonal variation of oxygen at mid depths in the Atlantic. These apparent periodic variations of oxygen within layers which have long since been shut off from contact with the atmosphere are assumed to result from the seasonal variations of oxygen in the surface water at the origin regions due to seasonal changes of phytoplankton growth; the effects of which continue to persist after a water mass has sunk into the depths and travelled great distances. The presence of this periodicity is

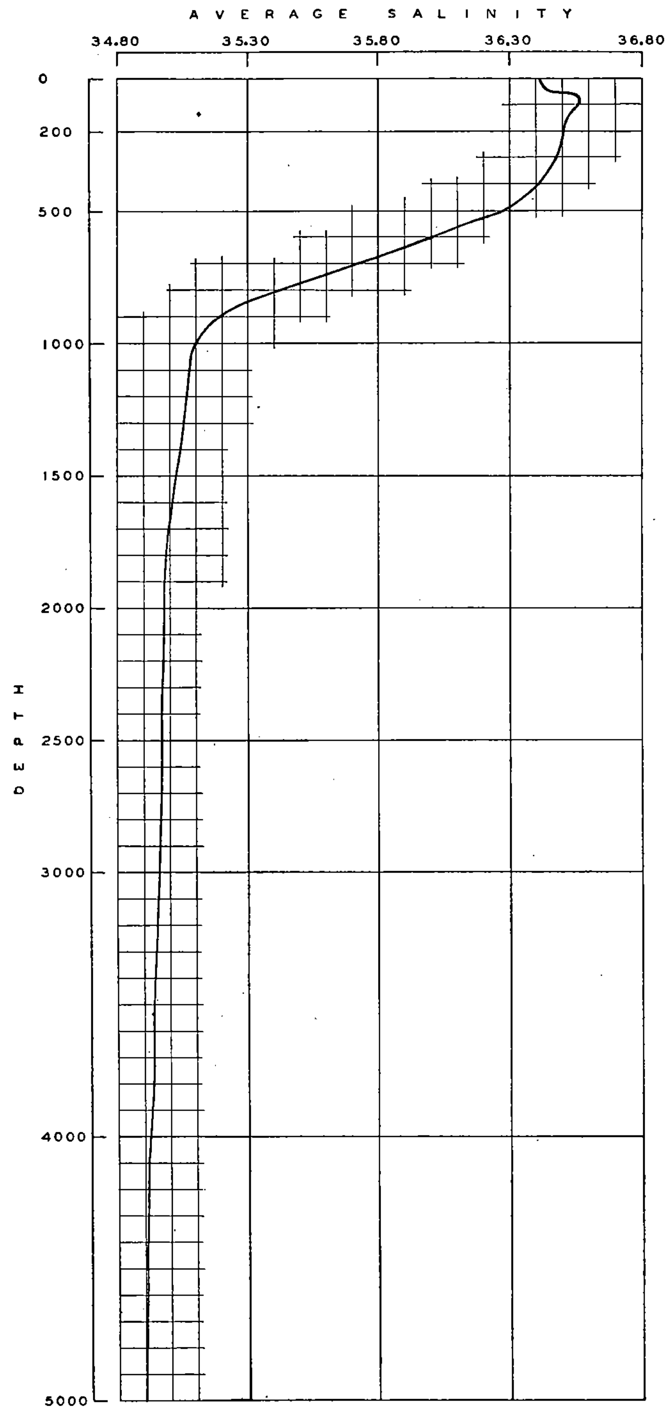

FIG. 9.—Mean vertical distribution of salinity at "Atlantis" station 2639.

made evident by a consecutive rising and falling of oxygen values in a horizontal direction (table 18). Reference to seasonal variation of oxygen in the deep layers of the Atlantic have been made by Wattenberg (1927), Deacon (1933), Seiwell (1934), and Wüst (1935), and in certain instances use has been made of these seasonal effects to calculate the velocities of deep currents (Deacon, 1933; Seiwell, 1934). Also, the presence of seasonal variations of oxygen in the deeper layers of the Atlantic affords a means, under certain ideal conditions, for calculating oxygen consumption in the deep water.

TABLE 18

Latitude	Oxygen Poor Layer O_2 cc/L	Difference	Latitude	Rich Underlying Layer O_2 cc/L	Difference
31° 30′	3.82		17° 42′	5.13	
		0.18			0.22
28° 48′	4.00		16° 00′	4.91	
		0.32			0.35
25° 30′	3.68		13° 18′	5.26	
		0.04			0.26
24° 30′	3.72		11° 24′	5.00	
		0.39			0.36
22° 24′	3.33		8° 42′	5.36	
		0.25			0.27
20° 42′	3.58		6° 36′	5.09	
		0.63			0.35
18° 06′	2.95		3° 12′	5.44	
		0.18			0.23
16° 30′	3.13		0° 54′	5.21	
		0.31			
14° 48′	2.82				
11° 06′	2.88				
		0.16			
8° 48′	3.04				
		0.02			
7° 42′	3.02				
		0.35			
5° 24′	3.37				
		0.12			
4° 00′	3.25				
		0.47			
1° 12′	3.72				

Horizontal variation of average oxygen content of oxygen poor layer and rich underlying layer along the 40th meridian. Scaled values. (From Seiwell, 1934).

And, since this is the only known method by which oxygen consumption from the deeper layers of the sea has been estimated, the question of the reality of seasonal oxygen variations in the deep water attains a high significance.

The horizontal variation of average oxygen values at mid depths in the mid north Atlantic is illustrated by the average oxygen contents of the oxygen poor layer (that part of the water less than 60 per cent saturated) and the rich underlying layer (to 2000 meters depth) along the 40th meridian between the equator and latitude 32° in table 18. Within the oxygen poor layer, oxygen is diminishing between latitudes 31° 30′ and 14° 48′ and increasing from latitudes 11° 06′ to 1° 12′; within the rich underlying layer oxygen is increasing between latitude 17° 42′ and 0° 54′. For both layers the difference between consecutive values is more than 0.10 cc per liter 90 per cent of the time, and more than 0.20 cc per liter 70 per cent of the time.

The question naturally arises concerning the possibility of a connection between short period fluctuations of oxygen content at fixed levels and the variations of average oxygen content of the oxygen poor and the underlying rich oxygen layers (illustrated by table 18). However, since the oxygen poor layer is not referred to any fixed level (bounded by isolines of 60 per cent saturation) the only way in which short period oscillations of the water column can alter the oxygen content of this layer is by producing convergences or divergences of the layer itself (due to vertical variations of amplitude and phase) in which a divergence of the oxygen poorest water will decrease the average oxygen content, and a convergence of oxygen poorest water will raise the average oxygen content or vice versa. Since such an effect is extremely unlikely to cause a variation as shown in table 18, especially over the entire horizontal distance between the equator and latitude 32°N, it need not be considered further.

On the other hand, the part of the rich underlying oxygen layer used for the analysis had its lower boundary referred to a fixed level at 2000 meters, while the upper boundary at 1200–1300 meters was marked by the 60 per cent isoline of relative saturation; and the possibility that periodic fluctuations in average oxygen content, of the magnitude observed in this layer (table 18), may be the result of sampling of a water column disturbed by short period oscillations may be tested by reference to the oxygen probability table (table 17). Thus, in the rich underlying layer differences between successive average maximum and average minimum values of less than 0.2 cc/liter did not occur and differences of 0.2 to 0.3 cc/liter occurred 57 per cent of the time, all other differences being greater than 0.3 cc/liter. On entering table 17 it is seen that, between depths of 1300 and 2000 meters, it is extremely unlikely that differences greater than 0.3 cc/liter are caused by vertical oscillations, whereas the expectant probability of differences from 0.2 to 0.3 cc/liter at indicated levels ranges from 0.036 to 0; and would be even less for average values. Hence, it appears unlikely that the maxima and minima fluctuations of the average oxygen content of the rich underlying oxygen layer of the magnitude observed are connected with short period oscillations of the water column.

The point may be raised that a comparison of conditions along the 40th meridian (between latitude 0 and 32°N) with station 2639 is not justified since the localities are widely separated and the short period variations of oxygen at both localities may not be comparable. Consideration must, thus, be given to a comparison of the preeminent conditions which primarily determine the magnitude of oxygen variation at individual levels resulting from vertical displacements of the water column, in an attempt to ascertain any significant disparity between the two regions. The conditions to be considered with respect to both localities are the vertical oxygen gradient and the order of magnitude of the vertical displacement.

For purposes of investigation the vertical oxygen gradient is expressed by the ratio:

$$\frac{\Delta O_2 \text{ cc/liter}}{\Delta Z \text{ meters}}$$

where, ΔO_2, is the difference in observed oxygen content between two observational depths separated by the vertical distance ΔZ. Values of the above ratio (approximately between 1200 and 2000 meters) for five "Atlantis" stations situated between 2° and 20°N (along the 40th meridian) are compared with those from station 2639 in table 19.

Examination of table 19 shows that the rate of change of oxygen with depth (between 1200 and 2000 meters) at five "Atlantis" stations along the 40th meridian between 2° and 20°N is of the same order of magnitude as that for station 2639. Consequently, to conclude that a vertical displacement of the water column of a given amplitude will produce a change of the same order of magnitude in the oxygen content at a fixed level along the 40th meridian in the southern part of the north Atlantic as it will for similar fixed levels at station 2639 seems reasonable.

TABLE 19

Station	2639 (35° 07′N, 66° 25′W)		
Interval	1200−1500	1500−1800	1800−2000
$\Delta O_2/\Delta Z$	22.3×10^{-4}	11.0×10^{-4}	4.0×10^{-4}
Station	1167 (19° 17′N, 41° 40′W)		
Interval	1107−1332	1332−1779	1779−2229
$\Delta O_2/\Delta Z$	24.5×10^{-4}	17.5×10^{-4}	4.0×10^{-4}
Station	1170 (14° 47′N, 40° 58′W)		
Interval	1173−1561	1561−1952	
$\Delta O_2/\Delta Z$	35.3×10^{-4}	14.6×10^{-4}	
Station	1173 (9° 57′N, 40° 55′W)		
Interval	1022−1401	1401−1768	1768−2247
$\Delta O_2/\Delta Z$	35.4×10^{-4}	22.1×10^{-4}	3.3×10^{-4}
Station	1175 (6° 50′N, 40° 25′W)		
Interval	1142−1978		
$\Delta O_2/\Delta Z$	23.5×10^{-4}		
Station	1178 (2° 02′N, 41° 18′W)		
Interval	1084−1626	1626−2183	
$\Delta O_2/\Delta Z$	31.4×10^{-4}	5.2×10^{-4}	

Data on the amplitude of vertical displacements of the water column below 1200 meters for the southern north Atlantic are not available. There is, however, no reason to suppose that they should be of significantly greater amplitude than those affecting the region characterized by station 2639 and in a very general way a comparison can be obtained on the order of magnitude of the vertical displacement of the deeper parts of the tropical Atlantic with station 2639 from the data obtained at "Meteor" station 241 (3° 50′S, 1° 05′E).[10] At the "Meteor" station the maximum variations of temperatures at various fixed levels obtained during a period of approximately 24 hours (Dec. 22 and 23, 1926) are given in table 20.

TABLE 20

Depth:	1200	1600	2000
"Meteor" 241	0.04	0.07	0.05
"Atlantis" 2639	0.50	0.16	0.09

At all levels below 1200 meters, temperature variations at "Atlantis" station 2639 were significantly greater than at "Meteor" station 241. In part, this is due to the difference in vertical position of the thermocline (it occupies a higher position at station 241), and, in part, due to the fact that "Atlantis" observations are more complete. However, there is nothing to indicate by these temperature values that the amplitude of vertical displacements for similar levels are any greater in the equatorial Atlantic than at the latitude of station 2639 (35°N).

On the basis of this discussion it appears likely that variations of the oxygen content at fixed levels resulting from the combined effect of vertical displacements of the water column and errors in the determination of oxygen are not of sufficient magnitude to produce the observed latitudinal horizontal variations in the average oxygen content of either the oxygen poor layer or the underlying oxygen rich layer in the southern north Atlantic along the 40th meridian. Therefore, if the observed horizontal variations of oxygen content (table 18) are not due to the effect of seasonal variations which have been impressed on the water mass at its origin region other causes must be sought.

[10] Defant, 1932.

APPENDIX

Analysis of the Hydrographic Wire Angle at Station 2639

In the present investigations the magnitude of the wire angle at the sea surface served as a means of estimating the spacing of the water bottles along the hydrographic wire in order to sample certain particular depths; the true depth of sampling being determined as indicated on page 6. A knowledge of the change of wire angle with depth for each lowering is also required for calculation of the falling velocity of the messenger (page 6).

During the period of observation at station 2639, the wire angle at the surface ranged from 0° to 48° during the 44 separate lowerings. This includes approximately the entire

TABLE 21

Surface Angle	Depth Interval	Average Wire Angle	Depth Interval	Average Wire Angle	Depth Interval	Average Wire Angle
6°	0–408	5.7°	408–1016	7.7°		
8°	0–511	7.7°	511–1015	8.1°		
8°	0–408	5.7°	408–1014	8.5°		
11°	0–324	10.9°	324– 866	10.2°	861–1087	8.9°
20°	0–323	11.8°	323– 862	11.5°	862–1074	12.6°
30°	0–384	20.4°	384– 977	17.6°		
33°	0–470	24.1°	470– 960	20.6°		
38°	0–416	36.2°	416– 893	29.4°		
43°	0–246	41.8°	246– 713	35.9°	713– 926	32.6°
45°	0–212	45.0°	212– 582	43.4°	582–1023	38.1°
47°	0–285	43.0°	285– 795	40.1°	795–1030	37.6°
47°	0–225	47.0°	225– 657	41.7°	657–1158	35.9°
49°	0–218	48.6°	218– 665	40.9°	665–1180	34.4°

range of surface wire angles usually encountered by "Atlantis," and, in a general way, the following brief analysis shows the relation between the inclination of the hydrographic wire at great depths as compared with that at the surface. These results may be considered representative for regions hydrographically similar to station 2639, which was marked by an absence of strong currents.

The following basic information was determined first:

1. Calculation of the average effective angles between depths determined by thermometers in the first lowering of the various series (A to W) making up station 2639.

2. Comparison of the visible surface angle with the average effective angle between surface and the topmost unprotected thermometer of the second and third lowering of each series (A to K).

3. Calculation of the average effective angle between the upper and lower unprotected thermometers of the second and the third lowerings of each series (A to K).

In the series A to K, which sampled to depths of approximately 5000 meters, the first lowering sampled between the approximate depths of 0 and 1000–1100 meters, the second lowering sampled between approximate depths of 1100 to 2750 meters, and the third lowering sampled between depths of 3000 to 5000 meters. Since it is customary to place unprotected thermometers on the upper and lower bottles of each deep lowering (in addition to one or two in between) it is possible to calculate the average effective wire angles, which existed between various levels, with a fair degree of accuracy. Hence, having determined the true depth the calculation of the average effective angle between any two depths is easily carried out. Thus:

$$\alpha = \cos^{-1} \frac{\text{true depth}}{\text{wire length}}.$$

Results of the calculation of the average effective wire angles for various first lowerings at station 2639 are compared with the visible surface angle in table 21.

When the surface wire angle of the hydrographic wire was 11° or less there was, for the first lowering, little change in angle with increasing depth as far as sampling extended (1000–1100 meters). When the surface wire angle was between 20° and 33° the inclination of the hydrographic wire at first declined rapidly with increasing depth (to depths of 350 to 450 meters), but still deeper the decline diminishes. With still larger surface wire angles (38° to 49°) there is little change in inclination to depths of around 250 meters, the decreases in wire inclination occurring chiefly in still deeper water.

In sampling between depths of 0 to 1000 meters significant errors may result from estimating the depth of sampling solely on the basis of the visible wire angle at the sea surface, unless that angle is 10° or less. The data in table 21 may be used as a guide in spacing the water bottles along the hydrographic wire when the surface angle is known and when working in regions hydrographically similar to station 2639.

Considering the second lowering (sampling began at 1100–1200 meters), a comparison of the surface wire angles with the average effective angle of the hydrographic wire to its uppermost water bottle reveals that for surface angles up to 11° the average inclination of the wire was only one to three degrees less, for surface wire angles between 17° and 25° the average inclination was about five degrees less and for surface wire angles of 32° or more the average inclination was generally fifteen degrees less. Considering the third or deepest lowerings it was found that for surface wire angles up to 25° the average inclination of the wire to the uppermost bottle (at approximately 3000 meters) was zero to five degrees less. Data corresponding to surface wire angles greater than 25° are not available for the third lowering.

Within that part of the water column sampled by the two deeper lowerings (the intervals between the upper and lower water bottles), the average inclination of the hydro graphic wire from the vertical for all the second lowerings ranged from 0° to 10.9° (average 7.2°), and for all third lowerings from 3.5° to 8.5° (average 6.7°). Hence, regardless of the visible angle of the hydrographic wire at the surface, the average inclination of the wire between the upper and lower bottles of these two deep lowerings was generally less than 10°.

Thus, in sampling at standard depths consideration should be given to the changing slope of the hydrographic wire with increasing depth; after measuring the visible surface angle the principal corrections are applied to: (1) spacing the water bottles along the hydrographic wire for the first lowering (which samples the upper 1100 meters), and (2) estimating the amount of wire to be reeled out for the two deeper lowerings. To space the water bottles along the wire within the second and third lowerings a correction for wire inclination of 7° may be applied when the surface wire angle is greater than 10°.

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